Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Visualizing Fractions

Model Fractions
Fractions Equivalent to One
Mixed Numbers and Improper Fractions
Equivalent Fractions

Manipulatives used:

Fraction circles
Fraction tiles

Model Fractions

Resources Needed:

Each student needs a worksheet, a set of fractions tiles, and a set of fraction circles.

Background Information:

Fractions are a very abstract idea to many students at this level. Students don't have a concrete model of fractions they can relate to, and so working with fractions becomes mere manipulation of symbols for no apparent reason. This activity helps students make the connection between a concrete fraction and the abstract concepts and symbols. The activity takes very little time, but the rewards are great.

Directions:

- In this quick activity the meaning of the numerator and denominator in a fraction are shown to correspond to parts of a whole. Students can complete this worksheet without using manipulatives. It is best done individually.
- Give each student a worksheet.
- Demonstrate for the class one "set" of fraction circles and one "set" of fraction tiles, (i.e., 3 thirds, 4 fourths, etc.). Show and explain how, for example, $\frac{1}{3}$ means 1 of the 3 equal pieces that together make one whole, and $\frac{2}{3}$ represents 2 of those pieces. Emphasize the meaning of fractions as parts of a whole.
- Have the students proceed through the worksheet on their own.
- When most students seem to have completed the worksheet, bring the class together again for discussion.
- Students can get additional practice naming fractions online at the National Library of Virtual Manipulatives website:
  - Fractions – parts of a whole http://nlvm.usu.edu/en/nav/frames_asid_102_g_2_t_1.html?from=topic_t_1.html
  - Fractions – naming http://nlvm.usu.edu/en/nav/frames_asid_104_g_1_t_1.html?from=topic_t_1.html
  - Fraction pieces http://nlvm.usu.edu/en/nav/frames_asid_274_g_3_t_1.html?open=activities&from=topic_t_1.html

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Model Fractions

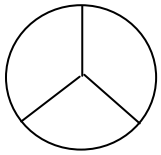
Fraction:

A fraction is written $\frac{a}{b}$

a is the **numerator** and b is the **denominator**.

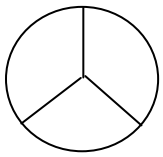
Fractions are a way to represent parts of a whole. The fraction $\frac{1}{3}$ means that one whole has been divided into 3 equal parts and each part is one of the three equal parts.

1) This circle that has been divided into 3 equal parts. Label each part $\frac{1}{3}$.



2) What does the fraction $\frac{2}{3}$ represent? This means the whole has been divided into 3 equal parts, and $\frac{2}{3}$ represents two of those three parts.

Shade two out of the three parts of this circle to represent $\frac{2}{3}$.

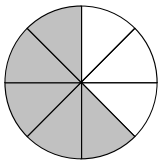


3) What fraction of this circle is shaded?

(a) How many parts are shaded? _____

(b) How many equal parts are there? _____

(c) The fraction of the circle that is shaded is $\frac{\square}{\square}$.

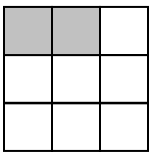


4) What fraction of this square is shaded?

(a) How many parts are shaded? ____

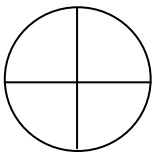
(b) How many equal parts are there? ____

(c) The fraction of the square that is shaded is $\frac{\square}{\square}$



5) To shade $\frac{3}{4}$ of the circle, shade _____ out of the _____ parts.

Shade $\frac{3}{4}$.



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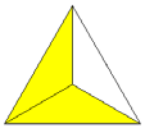
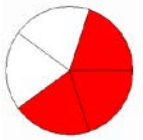
Model Fractions – Extra Practice

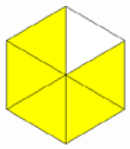
Name the fraction modeled by each figure.

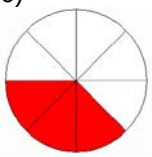
1)  _____

2)  _____

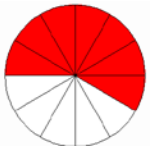
3)  _____

4)  _____

5)  _____

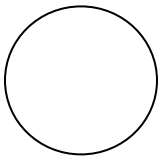
6)  _____

7)  _____

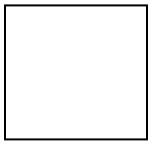
8)  _____

Model each fraction.

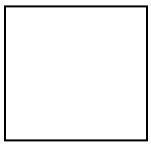
9) $\frac{1}{6}$



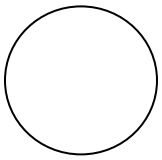
10) $\frac{5}{9}$



11) $\frac{4}{5}$



12) $\frac{7}{8}$



For more practice

- naming fractions, go to http://nlvm.usu.edu/en/nav/frames_asid_104_g_1_t_1.html?from=topic_t_1.html
- modeling fractions, go to http://nlvm.usu.edu/en/nav/frames_asid_102_g_2_t_1.html?from=topic_t_1.html

Fractions Equivalent to One

Resources Needed:

Each student needs a worksheet and a set of fractions tiles.

Background Information:

Fractions are a very abstract idea to many students at this level. Students don't have a concrete model of fractions they can relate to, and so working with fractions becomes mere manipulation of symbols for no apparent reason. This activity helps students make the connection between a concrete fraction and the abstract concepts and symbols. The activity takes very little time, but the rewards are great.

Directions:

- In this activity students use fraction tiles to model fractions equivalent to one. Students may work individually or with partners.
- Give each student a set of fraction tiles and a worksheet.
- Demonstrate for the class how to put all the fraction tiles together to make a rectangle of width one.

- Have the students proceed through the worksheet on their own or in their groups. Some students may need clarification when they attempt to answer the questions.
- When most students seem to have completed the worksheet, bring the class together again for discussion. You may want to ask the students for their answers to Exercise 5 and then list the 'patterns' they described in Exercise 6.
- The interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html shows a set of fraction tiles. Students can use it to verify, for example, that it takes fourteen $\frac{1}{14}$ pieces to make one.

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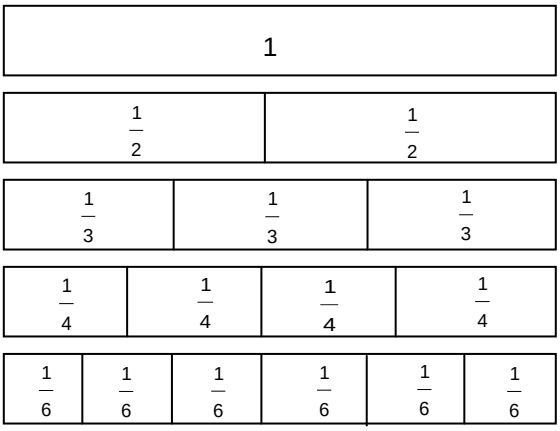
Fractions Equivalent to One

Fractions are often shown as parts of rectangles. Here, the whole is one long rectangle.



Set up your fraction tiles as shown in the diagram above.

1) How many of the $\frac{1}{2}$ tiles does it take to make 1 whole tile?

(a) It takes _____halves to make a whole. (b) Two out of two is 1 whole. $\frac{2}{2} =$ ____ .

2) How many of the $\frac{1}{3}$ tiles does it take to make 1 whole tile?

(a) It takes _____ thirds to make a whole. (b) Three out of three is 1 whole. $\frac{3}{3} =$ ____.

3) How many of the $\frac{1}{4}$ tiles does it take to make 1 whole tile?

(a) It takes _____fourths to make 1 whole. (b) Four out of four is 1 whole. $\frac{4}{4} =$ ____.

4) How many of the $\frac{1}{6}$ tiles does it take to make 1 whole tile?

(a) It takes _____ sixths. (b) Six out of six is 1 whole. $\frac{6}{6} =$ _____ .

5) What if the whole was divided into 24 equal parts? We don't have fraction tiles to represent this and it is too many to draw easily, but try to visualize it in your mind.

(a) How many $\frac{1}{24}$'s does it take to make 1? _____ (b) $\frac{\square}{24} = 1$

6) Do you see any pattern here? Describe the pattern you see.

Fractions Equivalent to One – Extra Practice

Use fraction tiles to answer these exercises.

You may want to use virtual fraction tiles on the interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html.

1) How many $\frac{1}{5}$'s does it take to make 1? _____

2) How many $\frac{1}{8}$'s does it take to make 1? _____

3) How many $\frac{1}{10}$'s does it take to make 1? _____

4) How many $\frac{1}{13}$'s does it take to make 1? _____

5) How many $\frac{1}{16}$'s does it take to make 1? _____

6) How many $\frac{1}{32}$'s does it take to make 1? _____

7) Fill in each numerator.

(a) $\frac{\square}{9} = 1$ (b) $\frac{\square}{12} = 1$ (c) $\frac{\square}{14} = 1$

8) Fill in each denominator.

(a) $\frac{8}{\square} = 1$ (b) $\frac{11}{\square} = 1$ (c) $\frac{15}{\square} = 1$

9) Fill in the missing part.

(a) $\frac{\square}{7} = 1$ (b) $\frac{20}{20} = \square$ (c) $\frac{25}{\square} = 1$

(d) $\frac{41}{41} = \square$ (e) $\frac{64}{\square} = 1$ (f) $\frac{\square}{100} = 1$

Mixed Numbers and Improper Fractions

Resources Needed:

Each student needs a worksheet and a set of fraction circles.

Background Information:

Fractions are a very abstract idea to many students at this level. Students don't have a concrete model of fractions they can relate to, and so working with fractions becomes mere manipulation of symbols for no apparent reason. These activities help students make the connection between a concrete fraction and the abstract concepts and symbols. The activities takes very little time, but the rewards are great.

Directions:

- In this activity students use fraction circles to model improper fractions and mixed numbers. Students should work in pairs, so they can model fractions larger than one.
- Give each student a set of fraction circles and a worksheet. Even though they work with a partner, all students should complete their own worksheets.
- Have the students work through the worksheet with their partners. Some students may need a hint to draw a third circle for question 2d.
- When most students seem to have completed the worksheet, bring the class together again for discussion. You may want to have the students share their answers to questions 8 and 11.
- Students can get more practice modeling improper fractions and visualizing how to convert between improper fractions and mixed numbers at the National Library of Virtual Manipulatives website.
  - Fraction pieces
    http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&from=topic_t_1.html

Mixed Numbers and Improper Fractions

1) Use fraction circles to make wholes, if possible, with the following pieces. Draw a sketch to show your result.
   (a) 2 halves (b) 6 sixths (c) 4 fourths (d) 5 fifths

2) Use fraction circles to make wholes, if possible, with the following pieces. Draw a sketch to show your result.
   (a) 3 halves (b) 5 fourths

   (c) 8 fifths (d) 7 thirds

When a fraction has the numerator smaller than the denominator, it is called a **proper** fraction. Its value is less than one. Fractions like $\frac{1}{2}$, $\frac{3}{7}$, and $\frac{11}{18}$ are proper fractions.

A fraction like $\frac{5}{4}$, $\frac{3}{2}$, $\frac{8}{5}$, or $\frac{7}{3}$ is called an **improper** fraction. Its numerator is greater than its denominator. Its value is greater than one.

Proper and Improper Fractions

The fraction $\frac{a}{b}$ is: $(b \neq 0)$

proper if $a < b$ or **improper** if $a \geq b$

3) Write as improper fractions.
   (a) 3 halves_____ (b) 5 fourths _____ (c) 8 fifths_____ (d) 7 thirds_____

4) Look back at your models in Exercise 2 and the improper fractions in Exercise 3. Which improper fraction in Exercise 3 could also be written as $1\frac{1}{4}$? _____

The number $1\frac{1}{4}$ called a **mixed number**; it consists of a whole number and a proper fraction.

Mixed Number

A **mixed number** is written $a\frac{b}{c}$ $c \neq 0$

A mixed number consists of a whole number *a* and a proper fraction $\frac{b}{c}$.

The model shows that $\frac{5}{4}$ has the same value as $1\frac{1}{4}$.

$\frac{5}{4} = 1\frac{1}{4}$

5) Write each improper fraction as a mixed number. You may want to refer to your models in Exercise 2.

(a) $\frac{3}{2}$ _____ (b) $\frac{5}{4}$ _____ (c) $\frac{8}{5}$ _____ (d) $\frac{7}{3}$ _____

6) Rewrite the improper fraction $\frac{11}{6}$ as a mixed number. Use fraction circles to find the result.

(a) Draw a sketch to show your answer. (b) $\frac{11}{6} =$ _____

7) Rewrite the improper fraction $\frac{17}{5}$ as a mixed number. Use fraction circles to find the result.

(a) Draw a sketch to show your answer. (b) $\frac{17}{5} =$ _____

8) Explain how you convert an improper fraction as a mixed number.

9) Rewrite the mixed number $1\frac{2}{3}$ as an improper fraction.

(a) Draw a sketch to show your answer. (b) $1\frac{2}{3} =$ _____

10) Rewrite the mixed number $2\frac{1}{4}$ as an improper fraction.

(a) Draw a sketch to show your answer. (b) $2\frac{1}{4} =$ ____

11) Explain how you convert a mixed number to an improper fraction.

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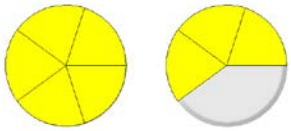
Mixed Numbers and Improper Fractions – Extra Practice

Use 2 sets of fraction circles to do these exercises.

You may want to use the fraction circles on the interactive website http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&from=topic_t_1.html.

Name each improper fraction. Then write each improper fraction as a mixed number.

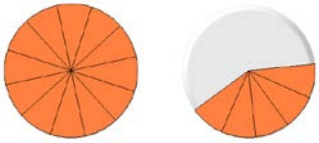
1)



(a) improper fraction _____

(b) mixed number _____

2)



(a) improper fraction _____

(b) mixed number _____

Draw a figure to model the following improper fractions. Then write each as a mixed number.

Improper fraction	Model	Mixed number
3) $\frac{7}{4}$		$\frac{7}{4} =$
4) $\frac{9}{5}$		$\frac{9}{5} =$
5) $\frac{17}{10}$		$\frac{17}{10} =$
6) $\frac{10}{3}$		$\frac{10}{3} =$

Draw a figure to model the following mixed numbers. Then write each as an improper fraction.

Mixed number	Model	Improper fraction
7) $1\frac{2}{5}$		$1\frac{2}{5} =$
8) $1\frac{1}{6}$		$1\frac{1}{6} =$
9) $1\frac{7}{12}$		$1\frac{7}{12} =$
10) $2\frac{3}{4}$		$2\frac{3}{4} =$

Equivalent Fractions

Resources Needed:

Each student needs a worksheet and a set of fractions tiles.

Background Information:

Many students that take this course have never really understood fractions. Often they just manipulate the symbols without any thoughts about their meaning, and as a result are just as likely to apply an incorrect procedure as the correct one. This activity helps students understand the concept of equivalent fractions and the procedure to find them; students will see how the abstract concepts and symbols relate to the concrete fraction tiles. This worksheet takes very little time, but the rewards are great.

Directions:

- Students may do this activity individually or in a small group.
- Give each student a set of fraction tiles and a worksheet. Be sure they all have an adequate amount of clear desk space to set out their fraction tiles.
- Guide them through the first part of the activity – finding how many fourths equal one-half. You may wish to use fraction tiles with a projector to demonstrate what it means to "exactly cover" the one-half tile.
- Let students continue with the worksheet on their own or in their groups.
- Discussion at the end will help reinforce the concepts. You may want to have students explain their answers to Exercises 6, 9, 13, and 14.
- The interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html shows a set of fraction tiles. Students drag a line across the set of tiles to see all the equivalent fractions.

Equivalent Fractions

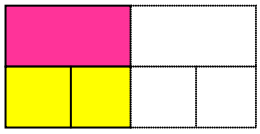
Equivalent Fractions
Equivalent fractions have the same value.

Use fraction tiles to do the following activity:

1) Take one of the $\frac{1}{2}$ tiles and set it on your workspace.

(a) How many fourths equal one-half?

Take the $\frac{1}{4}$ tiles and place them below the $\frac{1}{2}$ tile.



How many of the $\frac{1}{4}$ tiles exactly cover the $\frac{1}{2}$?_____

(b) Since _____ of the $\frac{1}{4}$ tiles cover the $\frac{1}{2}$ tile,

we see $\frac{\square}{4}$ is the same as $\frac{1}{2}$. $\frac{\square}{4}=\frac{1}{2}$

2) How many sixths equal one-half?

(a) How many of the $\frac{1}{6}$ tiles exactly cover the $\frac{1}{2}$ tile?_____

(b) Draw a sketch to show your result.

(c) Since _____ of the $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile,

we see $\frac{\square}{6}$ is the same as $\frac{1}{2}$. $\frac{\square}{6}=\frac{1}{2}$

3) How many eighths equal one-half?_____ $\frac{\square}{8}=\frac{1}{2}$

Draw a figure that demonstrates your answer.

4) How many tenths equal one-half?____ $\frac{\square}{10} = \frac{1}{2}$

Draw a figure that demonstrates your answer.

5) How many twelfths equal one-half?____ $\frac{\square}{12} = \frac{1}{2}$

Draw a figure that demonstrates your answer

6) Suppose you had bars marked $\frac{1}{20}$.

How many of them would it take to equal one-half?_____ $\frac{\square}{20} = \frac{1}{2}$

Take one of the $\frac{1}{3}$ bars and set it on your workspace.

7) How many sixths equal one-third?_____ $\frac{\square}{6} = \frac{1}{3}$

Draw a figure that demonstrates your answer.

8) How many twelfths equal one-third?_____ $\frac{\square}{12} = \frac{1}{3}$

Draw a figure that demonstrates your answer.

9) Suppose you had tiles marked $\frac{1}{30}$.

How many of them would it take to equal one-third? _____ $\frac{\square}{30} = \frac{1}{3}$

10) How many sixths equal two-thirds? _____ $\frac{\square}{6} = \frac{2}{3}$

Draw a figure that demonstrates your answer.

11) How many eighths equal three-fourths? _____ $\frac{\square}{8} = \frac{3}{4}$

Draw a figure that demonstrates your answer.

12) How many twelfths equal three-fourths? _____ $\frac{\square}{12} = \frac{3}{4}$

Draw a figure that demonstrates your answer.

13) Suppose you had tiles marked $\frac{1}{30}$.

(a) How many of them would it take to equal seven-tenths? _____ $\frac{\square}{30} = \frac{7}{10}$

(b) Explain how you got your answer.

14) Can you use twelfths to make a fraction equivalent to three-fifths? _____
Explain your reasoning.

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Equivalent Fractions – Extra Practice

Use fraction tiles to do these exercises. You may want to use virtual fraction tiles on the interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html

1) How many eighths equal one-fourth? _____ $\frac{\square}{8} = \frac{1}{4}$

Draw a figure that demonstrates your answer.

2) How many twelfths equal one-third? _____ $\frac{\square}{12} = \frac{1}{3}$

Draw a figure that demonstrates your answer.

3) How many tenths equal four-fifths? _____ $\frac{\square}{10} = \frac{4}{5}$

Draw a figure that demonstrates your answer.

4) How many sixteenths equal three-fourths? _____ $\frac{\square}{16} = \frac{3}{4}$

Draw a figure that demonstrates your answer.

5) How many fifteenths equal two-thirds? _____ $\frac{\square}{15} = \frac{2}{3}$

Draw a figure that demonstrates your answer.

6) How many fifteenths equal two-fifths? _____ $\frac{\square}{15} = \frac{2}{5}$

Draw a figure that demonstrates your answer.

7) How many twelfths equal six-eighths? _____ $\frac{\square}{12} = \frac{6}{8}$

Draw a figure that demonstrates your answer.

8) How many twelfths equal six-ninths? _____ $\frac{\square}{12} = \frac{6}{9}$

Draw a figure that demonstrates your answer.