Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Solving Equations

Subtraction Property of Equality
Division Property of Equality

Manipulatives used:

Small envelopes
Counters

Subtraction Property of Equality

Resources Needed:

Each student needs the worksheet, a small envelope and about 20 counters.

Background Information

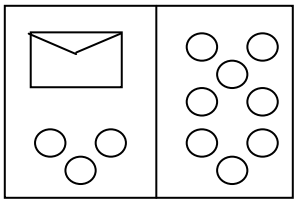
Students often think of solving equations as merely a mechanical process where you "move" numbers and variables across an equal sign. The "rules" don't make sense and so students often take steps that, even if they are algebraically correct, don't bring the equation closer to a solution. This exercise gives students a concrete model and guides their thought processes. By using the concrete model to develop the Subtraction Property of Equality, students are able to make the logical progression to the Addition Property of Equality.

Directions:

- This activity is best done with students in pairs.
- Do the first example together, using a projector, if possible. Set it up beforehand by putting 5 counters in an envelope. Then arrange the envelope and counters as shown in the figure. As you lead the class through it, make sure the students actually move the counters—this is important for the brain. When you have isolated the envelope, dump the counters out of it, to show that it does, indeed, contain 5 counters.
- Usually, students readily accept the natural progression from the model to the equation.
- Let the students continue on the worksheet. Some classes may not need to do all the exercises on the worksheet, but make sure all pairs of students do the last two exercises where they create equations for each other.
- Class discussion afterward will help reinforce the concepts. You may want to ask students to solve a few simple equations using the Subtraction Property of Equality with larger numbers.

Subtraction Property of Equality

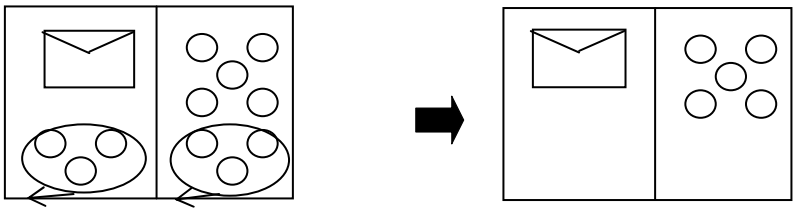
1) You are going to solve a puzzle. Use your envelopes and counters to recreate the picture below on your workspace. Both sides have the same number of counters, but some counters are "hidden" in the envelope. The goal is to discover how many counters are in the envelope.



(a) How many counters are in the envelope? _____counters are in the envelope.

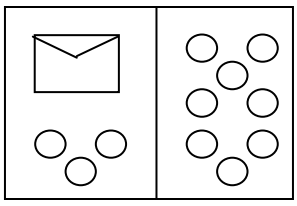
(b) What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope? List the steps here.

Perhaps you are thinking- the 3 counters at the bottom left can be matched with 3 on the right. Then I can take them away from both sides. That leaves five on the right-so there must be 5 counters in the envelope. Try this with your envelope and counters.



(c) Each side of the workspace models an expression and the line in the middle represents the equal sign, so we can write an algebraic equation from this model.

What algebraic equation is modeled by this picture?



_____ = _____

Let's write algebraically the steps we took to discover how many counters were in the envelope:

$x + 3 = 8$

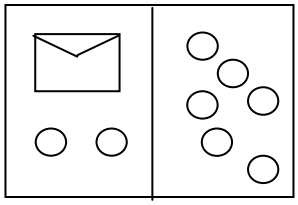
We took away three from each side. $x + 3 - \_\_\_ = 8 - \_\_\_$

And then we had ____ left. $x = 5$

(d) Check: $\_\_\_ + 3 = 8$

Five in the envelope plus three more equals eight!

2) Let's try this again! How many counters are in the envelope?



Use your envelope and counters to recreate this picture. Now, move the counters to find out how many counters are in the envelope.

(a) List the steps you took to find out how many counters were in the envelope.

(b) What algebraic equation is modeled by this picture? $x + \_\_\_\_\_ = \_\_\_\_\_$

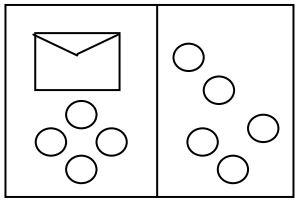
(c) We need to take away _____from each side. $x + 2 - \_\_\_\_ = 6 - \_\_\_\_$

(d) There are _____counters in the envelope! $x = \_\_\_\_$

(e) Check: $\_\_\_ + 2 = 6$

Four in the envelope plus two more does equal six!

3) How many counters are in this envelope?



Use your envelope and counters to recreate this picture. Move the counters to discover how many counters are in the envelope.

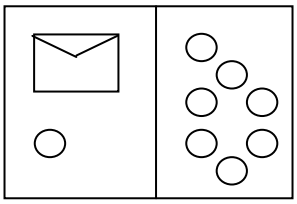
(a) Write the algebraic equation that is modeled by this picture. $x + \_\_\_\_\_ = \_\_\_\_\_$

(b) Take away _____ from each side. $x + 4 - \_\_\_\_ = 5 - \_\_\_\_$

(c) There are _____counters in the envelope! $x = \_\_\_\_$

(d) Check: $\_\_\_ + 4 = 5$

4) How many counters are in this envelope?



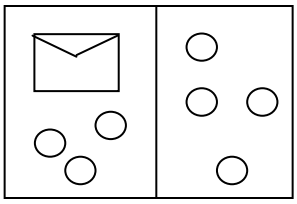
Use your envelope and counters to recreate this picture. Move the counters to find the number of counters in the envelope.

(a) Write the equation modeled by the envelope and counters. _____=_____

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

5) How many counters are in this envelope?



Use your envelopes and counters to recreate this picture. Move the counters as needed to find the number of counters in the envelope.

(a) Write the equation modeled by the envelope and counters. _____=_____

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

6) Model a similar equation for your partner. Have your partner figure out how many counters are in the envelope.
   (a) Sketch a picture of your model.

   (b) Show the algebra steps your partner took to find the number of counters in the envelope.

7) Have your partner model a similar equation for you. Figure out how many counters are in the envelope.
   (a) Sketch a picture of the model.

   (b) Show the algebra steps you took to find the number of counters in the envelope.

With these puzzles we have modeled a method for solving one kind of equation. To solve each equation, we used the Subtraction Property of Equality.

The Subtraction Property of Equality:
For any real numbers a, b, and c,
if $a = b$, then $a - c = b - c$.

When you subtract the same quantity from both sides of an equation, you still have equality!

Manipulative Mathematics **Name**_____

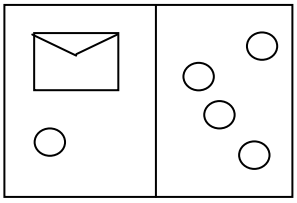
Subtraction Property of Equality – Extra Practice

#1-6: For each figure:

(a) Write the equation modeled by the envelopes and counters.

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

1)

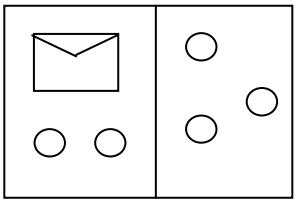


(a) Equation _____=_____

(b) Solution

Words	Algebra

2)

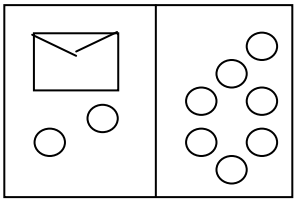


(a) Equation _____=_____

(b) Solution

Words	Algebra

3)

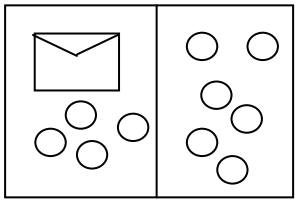


(a) Equation _____=_____

(b) Solution

Words	Algebra

4)

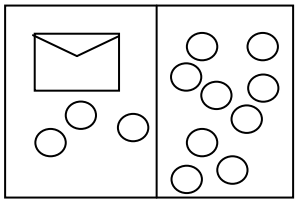


(a) Equation _____=_____

(b) Solution

Words	Algebra

5)

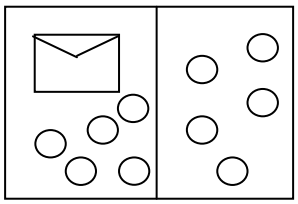


(a) Equation _____=_____

(b) Solution

Words	Algebra

6)



(a) Equation _____=_____

(b) Solution

Words	Algebra

#7-18: Solve each equation using the Subtraction Property of Equality.

7)
$$x+3=5$$
$$x+3-\_\_\_=5-\_\_\_$$
$$x=\_\_\_$$

8)
$$x+2=10$$
$$x+2-\_\_\_=10-\_\_\_$$
$$x=\_\_\_$$

9)
$$x+9=17$$
$$x+9-\_\_\_=17-\_\_\_$$
$$x=\_\_\_$$

10)
$$x+14=23$$
$$x+14-\_\_\_=23-\_\_\_$$
$$x=\_\_\_$$

11)
$$x+36=51$$
$$x+36-\_\_\_=51-\_\_\_$$
$$x=\_\_\_$$

12)
$$x+75=102$$
$$x+75-\_\_\_=102-\_\_\_$$
$$x=\_\_\_$$

13) $x+18=33$

14) $y+29=100$

15) $u+72=241$

16) $v+325=465$

17) $m+593=902$

18) $n+762=2014$

Division Property of Equality

Resources Needed:

Each student needs the worksheet, a few small envelopes and about 20 counters.

Background Information

Students often think of solving equations as merely a mechanical process where you "move" numbers and variables across an equal sign. The "rules" don't make sense and so students often take steps that, even if they are algebraically correct, don't bring the equation closer to a solution. This exercise gives students a concrete model and guides their thought processes. By using the concrete model to develop the Division Property of Equality, students are able to make the logical progression to the Multiplication Property of Equality.

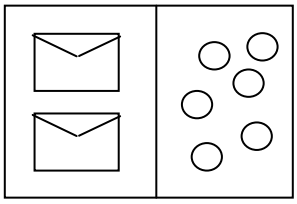
Directions:

- This activity is best done with students in pairs.
- Do the first example together, using a projector, if possible. Set it up beforehand with 2 envelopes containing 3 counters in each. Then arrange the envelope and counters as shown in the figure. As you lead the class through it, make sure the students actually move the counters—this is important for the brain. When you have separated the envelopes, dump the counters out of each one, to show that they do, indeed, each contain 3 counters.
- Usually, students follow the natural progression to the equation very well.
- Let the students continue on the worksheet. Some classes may not need to do all the exercises on the worksheet, but make sure all pairs of students do the last two exercises where they create equations for each other.
- Class discussion afterward will help reinforce the concepts. You may want to ask students to solve a few simple equations using the Division Property of Equality with bigger numbers.

Manipulative Mathematics **Name**_____

Division Property of Equality

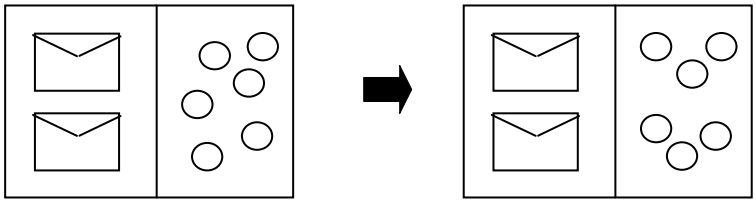
1) You are going to solve a puzzle. Use your envelopes and counters to recreate the picture below on your workspace. Both sides have the same total number of counters, but some counters are "hidden" in the envelopes. Both envelopes contain the same number of counters. The goal is to discover how many counters are in each envelope.



(a) How many counters are in each envelope? _____counters are in each envelope.

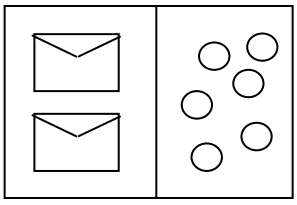
(b) What are you thinking? What steps are you taking in your mind to figure out how many counters are in each envelope? List the steps here.

Perhaps you are thinking that you have to separate the counters on the right side into 2 groups, because there are 2 envelopes. So 6 counters divided into 2 groups means there must be 3 counters in each envelope. Try this with your envelopes and counters.



(c) Each side of the workspace models an expression and the line in the middle represents the equal sign, so we can write an algebraic equation from this model.

What algebraic equation is modeled by this picture?



_____ = _____

(d) Let's write algebraically the steps we took to discover how many counters were in the envelope:

We divided both sides of the equation by _____,

So we have _____ in each envelope.

$$2x = 6$$

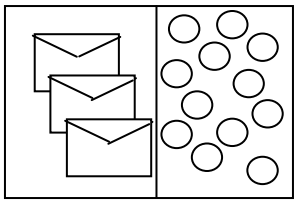
$$\frac{2x}{\square} = \frac{6}{\square}$$

$$x = 3$$

(e) Check: $2 \bullet \_\_\_ = 6$ Three counters in each of two envelopes equals six!

2) Here's another puzzle. How many counters are in each envelope?

Use your envelopes and counters to recreate this picture. Now, move the counters to find out how many counters are in each envelope.



(a) List the steps you took to find out how many counters are in each envelope.

(b) What algebraic equation is modeled by this picture? $\_\_\_x = \_\_\_\_\_$

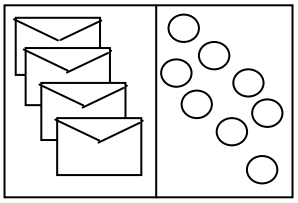
(c) We need to divide the counters into _____groups.

(d) Divide each side by _____. $\frac{3x}{\square} = \frac{12}{\square}$

(e) There are _____counters in each envelope! $x = \_\_\_\_$

(f) Check: $3 \bullet \_\_\_ = 12$

3) How many counters are in each envelope?
Use your envelopes and counters to recreate this picture. Move the counters to discover how many counters are in each envelope.



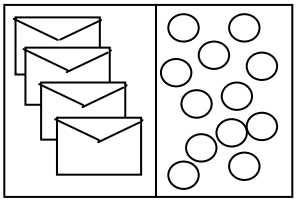
(a) Write the algebraic equation that would match this situation. $\_\_\_x = \_\_\_\_\_$

(b) Divide each side by_____. $\frac{4x}{\square} = \frac{8}{\square}$

(c) There are _____counters in each envelope! $x = \_\_\_\_$

(d) Check: $4 \bullet \_\_\_ = 8$

4) How many counters are in each envelope?
Use your envelopes and counters to recreate this picture.
Move the counters to find the number of counters in the envelope.

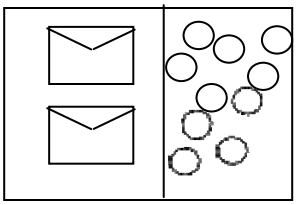


(a) Write the equation modeled by the envelopes and counters. ___ $x =$ _____

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

5) How many counters are in each envelope?
Use your envelopes and counters to recreate this picture.
Move the counters as needed to find the number of counters in the envelope.



(a) Write the equation modeled by the envelopes and counters. ___ $x =$ _____

(b) Show the steps you take, in words and algebra, to find the number of counters in the envelope.

Words	Algebra

6) Model a similar equation for your partner. Have your partner figure out how many counters are in each envelope.
   (a) Sketch a picture of your model.

   (b) Show the algebra steps your partner took to find the number of counters in each envelope.

7) Have your partner model a similar equation for you. Figure out how many counters are in each envelope.
   (a) Sketch a picture of the model.

   (b) Show the algebra steps you took to find the number of counters in each envelope.

With these puzzles we have modeled a method for solving one kind of equation. To solve each equation, we used the Division Property of Equality.

The Division Property of Equality

For any real numbers a,b,c, and $c \neq 0$,

if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

When you divide both sides of an equation by any non-zero number, you still have equality!

Manipulative Mathematics **Name**_____

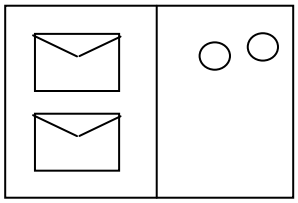
Division Property of Equality – Extra Practice

#1-6: For each figure:

(a) write the equation modeled by the envelopes and counters.

(b) show the steps you take, in words and algebra, to find the number of counters in each envelope.

1)

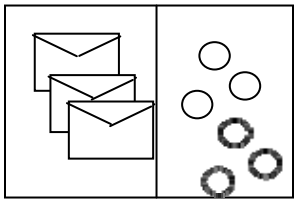


(a) Equation ___ $x =$ _____

(b) Solution

Words	Algebra

2)

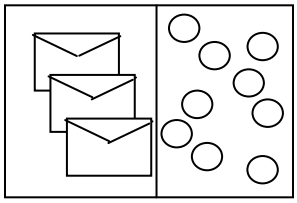


(a) Equation ___ $x =$ _____

(b) Solution

Words	Algebra

3)

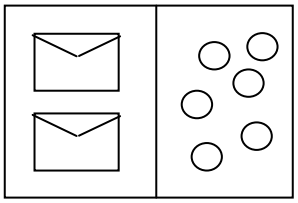


(a) Equation ___ $x =$ _____

(b) Solution

Words	Algebra

4)

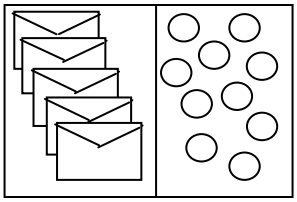


(a) Equation ___ $x =$ _____

(b) Solution

Words	Algebra

5)

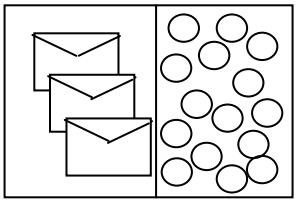


(a) Equation ___ $x =$ _____

(b) Solution

Words	Algebra

6)



(a) Equation ___ $x =$ _____

(b) Solution

Words	Algebra

#7-18: Solve each equation using the Division Property of Equality.

7) $2x = 16$

$\frac{2x}{\square} = \frac{16}{\square}$

$x =$ ___

8) $4x = 16$

$\frac{4x}{\square} = \frac{16}{\square}$

$x =$ ___

9) $8x = 16$

$\frac{8x}{\square} = \frac{16}{\square}$

$x =$ ___

10) $5x = 35$

$\frac{5x}{\square} = \frac{35}{\square}$

$x =$ ___

11) $9x = 54$

$\frac{9x}{\square} = \frac{54}{\square}$

$x =$ ___

12) $12x = 108$

$\frac{12x}{\square} = \frac{108}{\square}$

$x =$ ___

13) $7x = 42$

14) $11n = 165$

15) $19y = 38$

16) $25q = 375$

17) $80p = 800$

18) $101m = 909$