

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Number Lines

The Number Line Part 1-Counting Numbers and Whole Numbers

The Number Line Part 2-Integers

The Number Line Part 3-Fractions

Manipulatives used:

Paper tape

Markers

Resources Needed:

Each student needs the worksheet, a red, blue and black pen or marker and a three-foot strip of narrow paper. Adding machine paper works well.

Background Information:

By creating a number line and locating different sets of numbers on the line, students are making a concrete model of the number line. Students will visualize the relationship among number sets by outlining numbers with different symbols for each number set (counting, whole, integers). This kinesthetic activity accommodates the different learning styles of students. The number line model may also help students understand inequalities, especially when comparing negative numbers.

Directions:

Part 1-Counting numbers and whole numbers

- This activity may be done by individual students or together as a small group. Each student should create his own number line.
- Give each student the worksheet and a paper strip. Demonstrate folding the paper lengthwise and watch as the students fold theirs--there may be some confusion about how to fold the line. Then let the students follow the worksheet directions as you circulate through the classroom to offer individual guidance, as needed.
- Class discussion afterward will help reinforce the concepts of counting numbers and whole numbers.
- After students have completed this activity, tell them to save this number line in their notebooks, as they will use the same number line later to locate integers and fractions.

Part 2-Integers

- Integers will be added to the number line students created with counting numbers and whole numbers. Remind students to keep the unit the same for negatives as they used for positives.
- Then let the students proceed with the worksheet. Walk around the classroom to spot check student work.
- Class discussion afterward will help reinforce the concepts. Student number lines will now show counting numbers, whole numbers, and integers. While students have their number lines in front of them, you may wish to discuss ordering positive and negative numbers.
- After students have completed this activity, tell them to save this number line in their notebooks, as they will add fractions to this same number line later.

Part 3-Fractions

- After fractions have been introduced, they will be added to the number line. Many students seem to find this difficult.

- Demonstrate on the board how to divide the unit from 0 to 1 into various fractional units. Begin with halves and fourths. Continue, drawing a new unit interval for each new denominator, until the concept is clear.
- Then let the students proceed with the worksheet. Notice that the worksheet exercises do not use different denominators in the same unit interval. If there are student questions about locating, for example, $\frac{5}{3}$ when they already have $\frac{7}{4}$ marked (so the interval from 1 to 2 has been divided into fourths), draw a new number line.
- Class discussion afterward will help reinforce the concepts. Since students' number lines will have counting numbers, whole numbers, and integers marked, you may wish to introduce the term 'rational numbers' and explain how this number set relates to the others.
- Students can get additional practice locating fractions between 0 and 1 online at www.mathsisfun.com/numbers/fractions-match-frac-line.html. Given a fraction, students must decide how many intervals ('slices') to mark and then place the fraction at the correct mark. In addition to helping students locate proper fractions on the number line, it may also help students visualize fractions as parts of a whole.

Manipulative Mathematics
Number Line

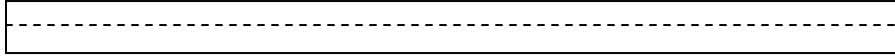
Name _____

The Number Line Part 1 -- Counting Numbers and Whole Numbers

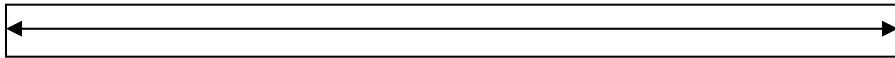
Counting numbers and whole numbers can be visualized by creating a **number line**.

1) To create your own number line:

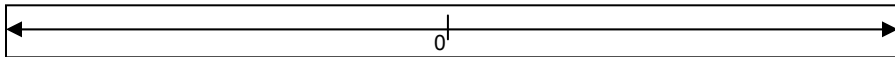
(a) Take a strip of paper about 3 feet long and fold it lengthwise to make a straight crease.



(b) Open the fold and draw a line in the crease. Put an arrow at each end of the line to indicate that the line continues.



(c) Mark a point at about the middle of the line. Label that point 0. This point is called the **origin**.



2) Choose a convenient unit and mark off several of these units to the right of 0. Pair these points with the numbers 1, 2, 3, 4, 5,and so on. When a number is paired with a point, we call it the **coordinate** of the point.



3) Draw a red triangle around each counting number.



4) Draw a blue circle around each whole number.



5) Notice that all the numbers on your number line except 0 are marked with both a triangle and a square. What conclusion can you draw from this?

6) In one corner of your strip make a "key" that explains the symbols around the numbers.



Counting numbers



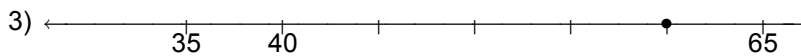
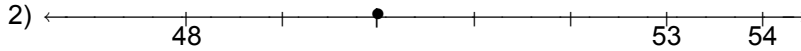
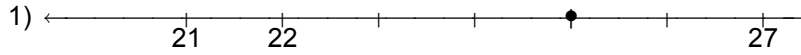
Whole numbers

7) Put your number line in your notebook for future use, so you can add more numbers to the number line as you proceed through this course.

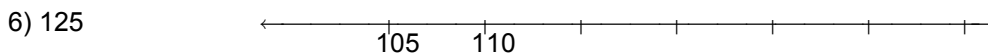
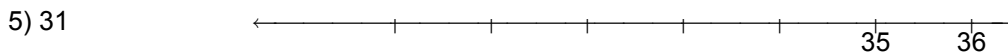
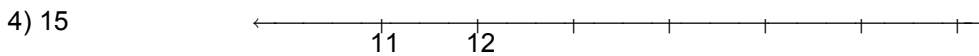
Manipulative Mathematics
Number Line Part I – Extra Practice

Name _____

Name the coordinate of each point.



Locate each point on the number line.



For each set of numbers identify (a) the counting numbers and (b) the whole numbers.

7) $0, \frac{1}{5}, 4, 7.5, 23, 199$ (a) _____ (b) _____

8) $0, \frac{3}{4}, 1, 5\frac{1}{2}, 16, 99.9, 250$ (a) _____ (b) _____

9) $0, \frac{2}{9}, 3.1, 6, 10\frac{1}{4}, 88, 132.5$ (a) _____ (b) _____

10) $0, 1, \frac{5}{2}, 5.2, 8, 24.99, 165, 200$ (a) _____ (b) _____

Number Line

The Number Line Part 2 -- Integers

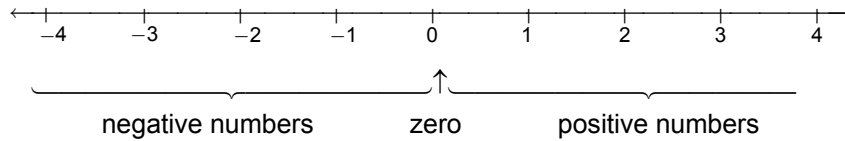
The number line you made in Part 1 started at 0. All the numbers you have worked with so far have been positive numbers, numbers greater than 0.



Now you need to expand your number line to include negative numbers, too. Negative numbers are numbers less than zero. So the negative numbers will be to the left of zero on the number line.

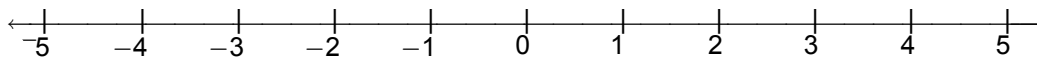
Get your number line out of your notebook and place it on your desk.

- 1) Mark off several units to the **left** of zero. Make sure your unit is the same size as the one you used on the positive side.
- 2) Now label -1 at the first point left of 0, then -2 at the next point to the left, and so on.



- 3) The arrows on both ends of the number line indicate that the numbers keep going forever.
 - (a) Is there a largest positive number? ____
 - (b) Is there a smallest negative number? ____
- 4) Is zero a positive or a negative number? _____ Numbers larger than zero are positive and numbers smaller than zero are negative. Zero is neither positive nor negative.
- 5) Locate and label the following points on this number line.

- (a) 2 (b) -1 (c) -4 (d) 5 (e) -5



The whole numbers and their opposites are called the **integers**.

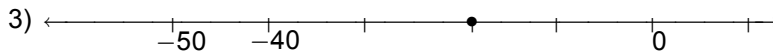
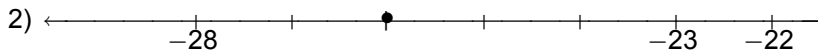
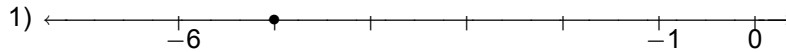
Integers: ... $-3, -2, -1, 0, 1, 2, 3, \dots$

- 6) Put a black square around each integer on your number line.
- 7) What do you notice about the integers, counting numbers and whole numbers on your number line?

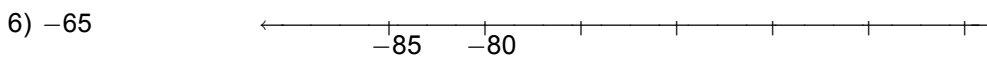
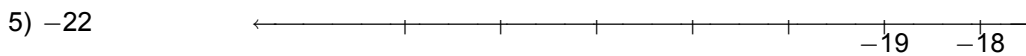
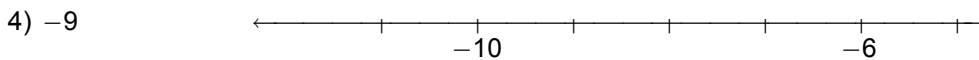
Manipulative Mathematics
Number Line Part 2 – Extra Practice

Name _____

Name the coordinate of each point.



Locate each point on the number line.



For each set of numbers identify the (a) counting numbers, (b) whole numbers, and (c) integers.

7) $-3, -\frac{1}{2}, 0, \frac{9}{10}, 5, 7.5, 32$ (a) _____ (b) _____ (c) _____

8) $-12, -\frac{3}{4}, 0, 2, 4.65, 29, 48\frac{1}{6}$ (a) _____ (b) _____ (c) _____

9) $-8.2, -3, -\frac{5}{9}, 0, 4, \frac{26}{3}, 99$ (a) _____ (b) _____ (c) _____

10) $-\frac{15}{4}, -2.5, -1, 0, \frac{4}{7}, 10, 28.1$ (a) _____ (b) _____ (c) _____

Number Line

The Number Line Part 3 -- Fractions

Now you are ready to include fractions on your number line. This will help you visualize fractions and understand their value. Take your number line out of your notebook and place it on your desk.

Our goal is to locate the numbers $\frac{1}{5}$, $\frac{4}{5}$, 3, $3\frac{1}{3}$, $\frac{7}{4}$, $\frac{9}{2}$, 5, and $\frac{8}{3}$ on the number line.

1) We'll start with the whole numbers 3 and 5 because they are the easiest to plot.

Put points to mark 3 and 5.



2) The proper fractions listed are $\frac{1}{5}$ and $\frac{4}{5}$.

(a) Proper fractions have value less than one. Between which two whole numbers are the proper fractions $\frac{1}{5}$ and $\frac{4}{5}$ located? They are between _____ and _____.

(b) Their denominators are both 5.

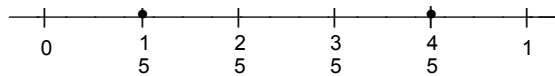
So into how many pieces do you need to divide the unit from 0 to 1? _____

How many marks will you need to divide the unit into that many pieces? _____

(c) Divide the unit from 0 to 1 into five equal parts, and label the marks,

consecutively, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

(d) Now put points to mark $\frac{1}{5}$ and $\frac{4}{5}$.



3) The only mixed number to plot is $3\frac{1}{3}$.

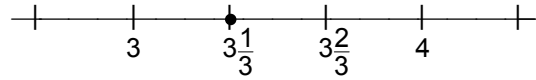
(a) Between which two whole numbers is $3\frac{1}{3}$? Remember that a mixed number is a whole

number plus a proper fraction, so $3\frac{1}{3} > 3$. Since it is greater than three, but not a whole

unit greater, $3\frac{1}{3}$ is between _____ and _____.

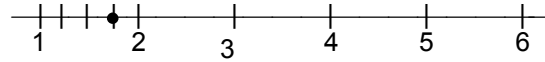
(b) Divide that portion of the number line into _____ equal pieces (thirds) by making _____ marks.

(c) Plot $3\frac{1}{3}$ at the first mark.

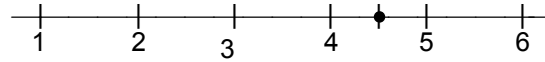


4) Finally, look at the improper fractions $\frac{7}{4}$, $\frac{9}{2}$, $\frac{8}{3}$. Locating these points will be easier if you change each of them to a mixed number.

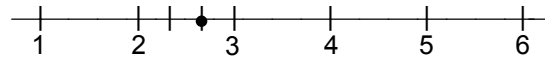
(a) $\frac{7}{4} =$ _____



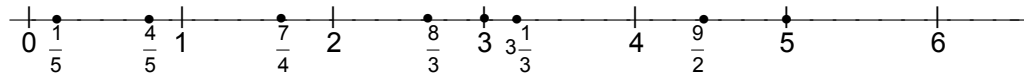
(b) $\frac{9}{2} =$ _____



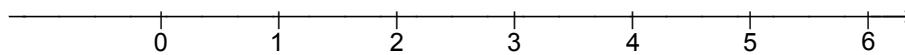
(c) $\frac{8}{3} =$ _____



5) Here is the number line with all the points ($\frac{1}{5}$, $\frac{4}{5}$, 3, $3\frac{1}{3}$, $\frac{7}{4}$, $\frac{9}{2}$, 5, and $\frac{8}{3}$) plotted. Verify that your number line looks the same.



6) Locate and label the fractions $\frac{3}{4}$, $\frac{4}{3}$, $\frac{5}{3}$, $4\frac{1}{5}$, $\frac{7}{2}$ on the number line below.



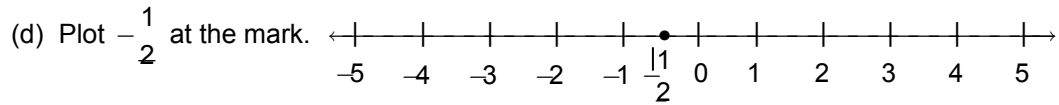
Now let's locate some negative fractions.

7) We'll locate $-\frac{1}{2}$ first. Remember that negative numbers are opposites of positive numbers, so $-\frac{1}{2}$ is the opposite of $\frac{1}{2}$.

(a) Since $\frac{1}{2}$ is between the two whole numbers _____ and _____, $-\frac{1}{2}$ is between the two integers _____ and _____.

(b) Into how many pieces do we need to divide the unit between 0 and -1 ? _____

(c) Divide that portion of the number line into _____ equal pieces (halves) by making _____ marks.



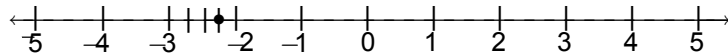
8) Now let's locate $-2\frac{1}{4}$ on a number line.

(a) Think about $2\frac{1}{4}$ first. It is located between the whole numbers _____ and _____.

(b) So $-2\frac{1}{4}$ is between _____ and _____.

(c) Into how many equal pieces do we need to divide that unit? _____

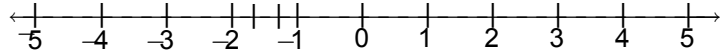
(d) Plot $-2\frac{1}{4}$ at the first mark.



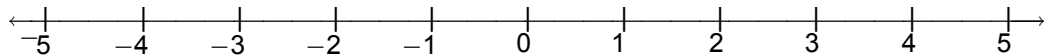
9) Locating $-\frac{5}{3}$ on a number line will be easier if you first change it to a mixed number.

(a) $-\frac{5}{3} =$ _____. It is between _____ and _____.

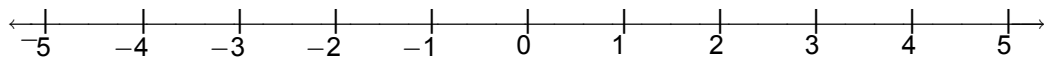
(b) Plot $-\frac{5}{3}$.



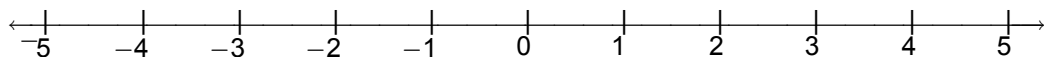
10) Locate and label the fractions $\frac{2}{3}$ and $-\frac{2}{3}$ on the number line below.



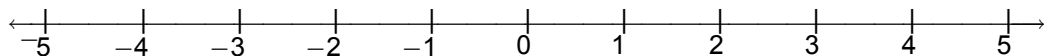
11) Locate and label the fractions $-\frac{9}{2}$ and $-\frac{9}{4}$ on the number line below.



12) Locate and label the fractions $\frac{7}{3}$, $-3\frac{3}{4}$, $3\frac{1}{3}$, and $-\frac{8}{5}$ on the number line below.



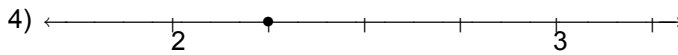
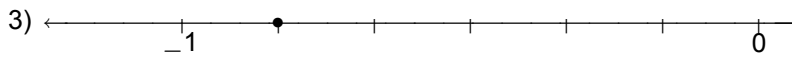
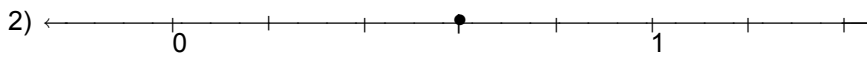
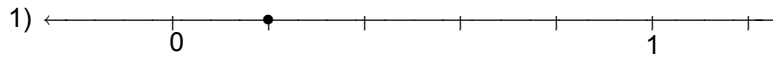
13) Locate and label the fractions $\frac{1}{3}$, $-\frac{5}{4}$, $-\frac{7}{4}$, $2\frac{3}{5}$, and $-3\frac{1}{2}$ on the number line below.



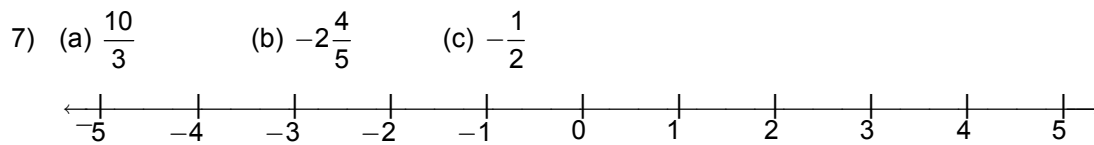
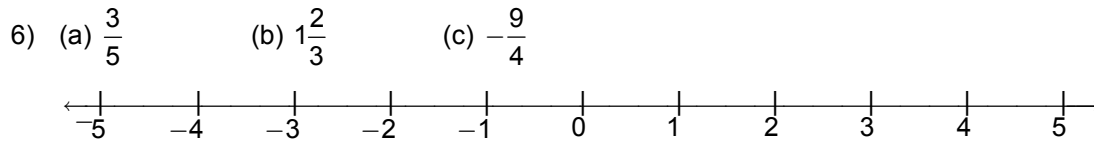
Manipulative Mathematics
Number Line Part 3 – Extra Practice

Name _____

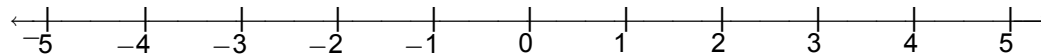
Name the coordinate of each point.



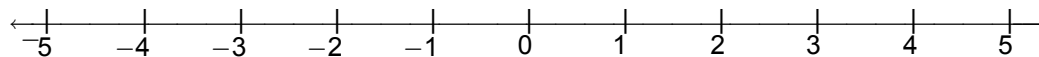
Locate and label each point on the number line.



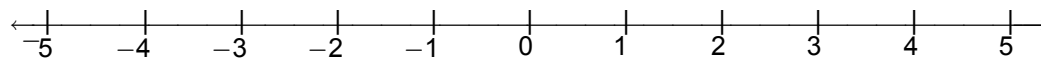
8) (a) $-3\frac{3}{4}$ (b) $\frac{1}{3}$ (c) $\frac{15}{4}$



9) (a) $-\frac{5}{4}$ (b) $4\frac{2}{3}$ (c) $-\frac{4}{5}$



10) (a) $\frac{5}{8}$ (b) $-3\frac{1}{4}$ (c) $-\frac{14}{3}$



You can do more practice locating fractions on the number line at the website
<http://www.mathsisfun.com/numbers/fractions-match-frac-line.html>.