Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Multiplying and Dividing Fractions

Model Fraction Multiplication
Model Fraction Division

Manipulatives used:

Fraction circles
Fraction tiles

Model Fraction Multiplication

Resources Needed:

Each student needs the worksheet and two highlighters or colored pencils.

Background Information:

Many students who take this course have never been comfortable with fractions. They may have never made the connection between a concrete model of a fraction as part of a whole and the abstract concept and symbols. Fraction operations are merely rules that make no sense and thus are easily confused. By modeling the operations on fractions, students begin to understand how and why the procedures work, and may apply them more consistently. Working with models makes many students feel more competent and in control since it helps the procedures make sense.

The model of fraction multiplication used in this activity translates the expression $\frac{1}{2}\bullet\frac{3}{4}$ as "one-half of three-fourths". This is consistent with how fraction multiplication is applied in many real-life situations, such as two people sharing three-fourths of a pizza.

Directions:

- You may wish to review the definition of a fraction as part of a whole with your students before starting this activity. Understanding that concept is a prerequisite for this activity.
- This activity may be done by students in small groups or as individuals.
- Give each student the worksheet. Each student will need to use two highlighters or colored pencils.
- Demonstrate the first example of modeling multiplication, $\frac{1}{2}\bullet\frac{3}{4}$. Have students work along with you to complete parts (a) through (e).
- Let the class proceed through the worksheet activities. You may want to make sure everyone is actually modeling the multiplications rather than just writing the answers, as this is important to developing conceptual understanding.
- Discussion at the end of this activity will help reinforce the concepts. Make sure everyone can use the definition of Fraction Multiplication given in Exercise 5.
- The rectangle model of fraction multiplication can be found at the website http://nlvm.usu.edu/en/nav/frames_asid_194_g_2_t_1.html?from=search.html?qt=multiply+fractions.

Name_____

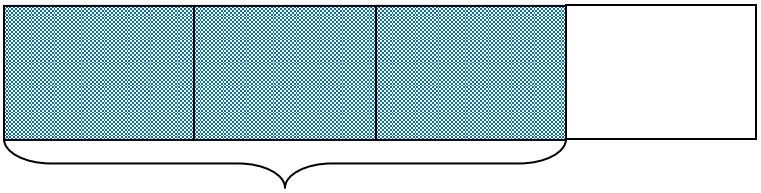
Model Fraction Multiplication

When you multiply fractions, do you need a common denominator? Do you take the reciprocal of one of the fractions? What are you supposed to do and how are you going to remember it? A model may help you understand multiplication of fractions.

1) Model the product $\frac{1}{2} \bullet \frac{3}{4}$.

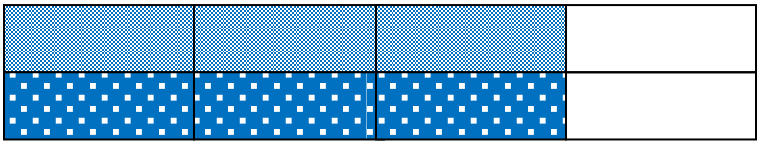
(a) To multiply $\frac{1}{2}$ and $\frac{3}{4}$, let's think "$\frac{1}{2}$ of $\frac{3}{4}$".

(b) First, we draw a rectangle to represent one whole. We divide it vertically into 4 equal parts, and then shade in three of the parts to model $\frac{3}{4}$.



We have shaded in $\frac{3}{4}$ of the rectangle.

(c) Now, we divide the rectangle horizontally into two equal parts to divide the whole into halves. Then we double-shade $\frac{1}{2}$ of what was already shaded.



(d) Into how many equal pieces is the rectangle divided now? _____

(e) How many of these pieces are double-shaded? _____

We double-shaded 3 out of the 8 equal pieces, $\frac{3}{8}$ of the rectangle. So $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$.

We showed that

$$\frac{1}{2} \bullet \frac{3}{4} = \frac{3}{8}$$

Notice –

multiplying the numerators $1 \bullet 3 = 3$

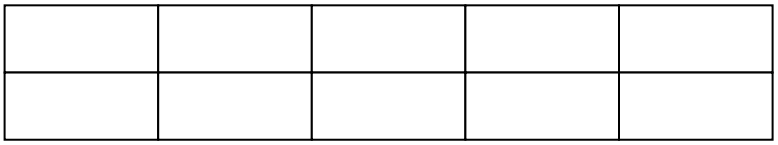
multiplying the denominators $2 \bullet 4 = 8$

2) Model the product $\frac{1}{2} \bullet \frac{3}{5}$.

(a) To multiply $\frac{1}{2}$ and $\frac{3}{5}$, think "$\frac{1}{2}$ of_____".

(b) First shade in $\frac{3}{5}$ of the rectangle.

(c) Now double-shade $\frac{1}{2}$ of what was already shaded.



(d) Into how many equal pieces is the rectangle divided now? _____

(e) How many pieces are double-shaded? _____

(f) What fraction of the rectangle is double-shaded? _____

(g) So $\frac{1}{2}$ of $\frac{3}{5}$ is _____.

You have shown that

$$\frac{1}{2} \bullet \frac{3}{5} = \frac{3}{10}$$

Notice –

multiplying the numerators $1 \bullet 3 = 3$

multiplying the denominators $2 \bullet 5 = 10$

3) Use a rectangle to model each product. Sketch a diagram to illustrate your model.

(a) $\frac{1}{2} \bullet \frac{1}{3}$

$\frac{1}{2} \bullet \frac{1}{3} =$ ____

(b) $\frac{1}{2}\bullet\frac{1}{4}$

$\frac{1}{2}\bullet\frac{1}{4}=$ ____

(c) $\frac{1}{3}\bullet\frac{1}{4}$

$\frac{1}{3}\cdot\frac{1}{4}=$ ___

(d) $\frac{1}{3}\bullet\frac{2}{3}$

$\frac{1}{3}\bullet\frac{2}{3}=$ ___

(e) $\frac{2}{3}\bullet\frac{4}{5}$

$\frac{2}{3}\bullet\frac{4}{5}=$ ___

4) Look at each of your models and answers in Question 3.

(a) If you multiply numerators and multiply denominators, do you get the same result as you did from the model? _____

(b) Explain in words how to multiply two fractions.

5) The definition of fraction multiplication is given in the box below.

Fraction Multiplication

If a,b,c, and d are numbers where $b \neq 0 \text{ and } d \neq 0$, then $\frac{a}{b}\bullet\frac{c}{d}=\frac{ac}{bd}$.

To multiply fractions, multiply the numerators and multiply the denominators.

Use the definition of fraction multiplication to multiply $\frac{5}{12}\bullet\frac{7}{3}$

(a) Identify a,b,c, and d .

(b) Multiply the fractions.

Model Fraction Multiplication – Extra Practice

Use a rectangle to model each multiplication. Sketch your model and write the product.

You can practice using rectangles to model fraction multiplication online at the website http://nlvm.usu.edu/en/nav/frames_asid_194_g_2_t_1.html?from=search.html?qt=multiply+fractions.

1) $\frac{1}{2} \bullet \frac{1}{6}$

2) $\frac{1}{2} \bullet \frac{1}{8}$

3) $\frac{1}{3} \bullet \frac{1}{3}$

4) $\frac{1}{4} \bullet \frac{1}{4}$

5) $\frac{1}{2} \bullet \frac{5}{8}$

6) $\frac{1}{2} \bullet \frac{5}{6}$

Multiply.

7) $\frac{2}{3} \bullet \frac{2}{5}$

8) $\frac{2}{5} \bullet \frac{4}{5}$

9) $\frac{3}{5} \bullet \frac{7}{8}$

10) $\frac{3}{4} \bullet \frac{5}{8}$

Model Fraction Division

Resources Needed:

Each student needs the worksheet and a set of fractions tiles.

Background Information:

Many students who take this course have never been comfortable with fractions. They may have never made the connection between a concrete model of a fraction as part of a whole and the abstract concept and symbols. Fraction operations are merely rules that make no sense and thus are easily confused. By modeling the operations on fractions, students begin to understand how and why the procedures work, and may apply them more consistently. Working with models makes many students feel more competent and in control since it helps make sense of the procedures.

The division model used in this activity is the 'goes into' model—for example, how many times does one-sixths go into one-half. This translates easily into 'how many one-sixths are there in one-half'. This model is consistent with division of whole numbers, a more familiar concept to most students, and may be presented as an extension of that idea.

Directions:

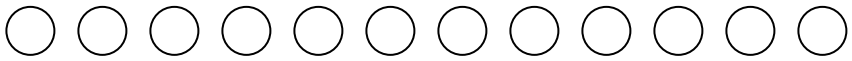
- This activity may be done by students in small groups or as individuals.
- Give each student the worksheet and a set of fraction tiles.
- Model questions 1 and 2 from the worksheet together with the class. You may also want to talk about money to make fraction division real – how many quarters are in a half-dollar?
- Let the class proceed through the rest of the worksheet activities. You may want to make sure everyone is modeling the divisions rather than just writing the answers, as this is important to developing conceptual understanding.
- Discussion at the end of this activity will help reinforce the concepts. Question 8 is the only division that doesn't come out to be a whole number—this can lead to a good discussion of the size of the two fraction relative to each other. You may also want students to share their responses to Exercises 9 and 10.
- You might find it helpful to show your students the interactive fraction tile graphic at http://www.mathsisfun.com/numbers/fraction-number-line.html. You can move the slider and count, for example, how many twelfths are in three-fourths.

Model Fraction Division

Model Fraction Division

1) Why is $12 \div 3 = 4$? Let's model this with counters.

   (a) How many groups of 3 counters can be made from the 12 shown below?



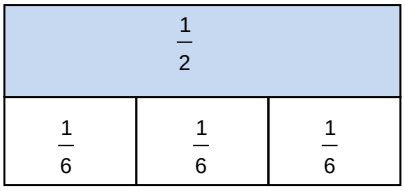
   (b) Draw a circle around each group of 3 counters. How many groups of 3 counters do you have? _____

   (c) There are _____ groups of 3 counters. In other words, there are _____3s in 12. So, $12 \div 3 =$ _____ .

What about dividing fractions? Get out your fraction tiles and let's see!

2) To model the quotient $\frac{1}{2} \div \frac{1}{6}$ with fraction tiles we want to see how many sixths there are in one-half.

   (a) Line up your half and sixth fraction tiles as shown below.



   (b) How many $\frac{1}{6}$s are in $\frac{1}{2}$? _____

   (c) $\frac{1}{2} \div \frac{1}{6} =$ _____

3) Model the quotient $\frac{1}{4} \div \frac{1}{8}$ with fraction tiles.

   Use your fourth and eighth fraction tiles to find out how many eighths there are in one fourth.

   (a) Draw a sketch of your result here.

   (b) There are _____ $\frac{1}{8}$s in $\frac{1}{4}$.

   (c) So $\frac{1}{4} \div \frac{1}{8} =$ _____

4) Model the quotient $\frac{1}{3} \div \frac{1}{6}$ with fraction tiles

Use your third and sixth fraction tiles to find out how many sixths there are in one third

a) Draw a sketch of your result here.

b) There are _____ $\frac{1}{6}$s in $\frac{1}{3}$.

c) So $\frac{1}{3} \div \frac{1}{6} =$ _____

5) Model the quotient $\frac{1}{2} \div \frac{1}{8}$ with fraction tiles

Use your half and eighth fraction tiles to find out how many eighths there are in one half.

a) Draw a sketch of your result here.

b) There are _____ $\frac{1}{8}$s in $\frac{1}{2}$

c) So $\frac{1}{2} \div \frac{1}{8} =$ _____

Model a Whole Number Divided by a Fraction

6) Use fraction bars to model the quotient $2 \div \frac{1}{4}$

(a) How many $\frac{1}{4}$s are there in 2?

1				1			
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(b) There are _____ $\frac{1}{4}$s in 2, so $2 \div \frac{1}{4} =$ _____

(c) Let's think of this example another way—in terms of money. We often read $\frac{1}{4}$ as 'one quarter', so you can think of $2 \div \frac{1}{4}$, as asking "how many quarters are there in two dollars?" We know that \$1 is 4 quarters, so how many quarters are in \$2? _____

(d) So, $2 \div \frac{1}{4} =$ _____ .

7) Use fraction tiles to model the following. Sketch a diagram to illustrate your model.

a) $2 \div \frac{1}{3}$ b) $3 \div \frac{1}{2}$

$2 \div \frac{1}{3} =$ _____ $3 \div \frac{1}{2} =$ _____

Using fraction tiles in exercise 2, we showed that $\frac{1}{2} \div \frac{1}{6} = 3$. Notice that $\frac{1}{2} \bullet \frac{6}{1} = 3$ also. How does $\frac{6}{1}$ relate to $\frac{1}{6}$? They are reciprocals! To divide fractions, we multiply the first fraction by the reciprocal of the second. This leads to the following definition.

Fraction Division

If a,b,c, and d are numbers where $b \neq 0, c \neq 0$ and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$

8) Use the Fraction Division definition above to find the quotient $\frac{5}{7} \div \frac{3}{8}$.

(a) Identify the numbers that correspond to a,b,c, and d .

(b) Divide the fractions.

9) Explain in words how to divide two fractions.

10) Explain in words how to divide a whole number by a fraction.

Model Fraction Division – Extra Practice

Use fraction tiles to model each division. Sketch your model and write the quotient.

You may want to use the fraction tiles shown at the interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html.

1) $\frac{1}{2} \div \frac{1}{10}$

2) $\frac{1}{2} \div \frac{1}{12}$

3) $\frac{1}{3} \div \frac{1}{12}$

4) $\frac{1}{4} \div \frac{1}{12}$

5) $\frac{3}{4} \div \frac{1}{8}$

6) $\frac{2}{5} \div \frac{1}{10}$

Divide.

7) $\frac{5}{8} \div \frac{1}{6}$

8) $\frac{5}{6} \div \frac{1}{8}$

9) $\frac{2}{5} \div \frac{1}{2}$

10) $\frac{3}{10} \div \frac{1}{3}$