Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Adding and Subtracting Fractions

Model Fraction Addition
Model Fraction Subtraction
Model Finding the Least Common Denominator

Manipulatives used:

Fraction circles
Fraction tiles

Model Fraction Addition

Resources Needed:

Each student needs the worksheet and a set of fractions circles.

Background:

Many students who take this course have never been comfortable with fractions. Fraction addition and subtraction are especially problematic. Students don't understand why adding or subtracting across numerators and/or denominators is incorrect. Working with concrete models helps students see why a common denominator is needed.

Directions:

- This activity may be done by students individually or in small groups. The addition and subtraction worksheets may be done at separate times.
- Give each student a set of fraction circles and the worksheet.
- Demonstrate the example of modeling $\frac{1}{4}+\frac{2}{4}$ using quarters. Many students do not think of quarters as fractions of dollars. It is important that they see that relationship and are able to use to it as a reference point for fraction addition.
- Then use fraction circles to model the same addition, $\frac{1}{4}+\frac{2}{4}$.
- Let the class proceed through the worksheet activities.
- When most of the students have completed the worksheet, bring the whole class together. Discussion at the end of this activity will help reinforce the concepts. Since the goal is to have students understand addition of fractions with common denominators, you may want to accept answers that are not simplified, such as $\frac{6}{8}$.
- You may want to show your students how to use the fraction circles on the website http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

Model Fraction Addition

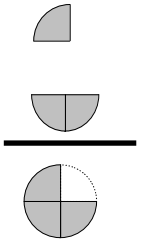
How many quarters are pictured above? One quarter plus 2 quarters equals 3 quarters. Quarters? Remember, quarters are really fractions of a dollar; "quarter" is another word for "fourth". So the picture of the coins shows that $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$.

Let's use fraction circles to model addition of fractions for the same example, $\frac{1}{4}+\frac{2}{4}$.

Start with one $\frac{1}{4}$ piece.

Add two more $\frac{1}{4}$ pieces.

The result is $\frac{3}{4}$.



$$\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$$

So, $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$.

1) Use fraction circles to model the sum $\frac{3}{8}+\frac{2}{8}$.

   (a) Take three $\frac{1}{8}$ pieces. Add two more $\frac{1}{8}$ pieces. How many $\frac{1}{8}$ pieces do you have? ___

   (b) Sketch your model here.

   (c) You have five eighths. $\frac{3}{8}+\frac{2}{8}=$ _____

2) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

   (a) $\frac{1}{3}+\frac{1}{3}=$ _____ .

   (b) $\frac{1}{6}+\frac{4}{6}=$ _____

   (c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

3) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{1}{5}+\frac{3}{5}=$ _____ (b) $\frac{2}{5}+\frac{2}{5}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

4) Use fraction circles to model the following. Sketch a diagram to illustrate your model**.**

(a) $\frac{3}{8}+\frac{4}{8}=$ _____ (b) $\frac{1}{8}+\frac{4}{8}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

5) A common error made by students when adding fractions is to add the numerators and add the denominators (much like we multiply numerators and multiply denominators when multiplying fractions). Use a model to see why this does not work for addition!

(a) Model $\frac{1}{5}+\frac{1}{5}$. Sketch a diagram to illustrate your model.

(b) Did the fifths change to another size piece?_____ Did they change to $\frac{1}{10}$ pieces?_____

(c) $\frac{1}{5}+\frac{1}{5}=$ _____

These examples show that to add the same size fraction pieces—that is, fractions with the same denominator—you just add the number of pieces. So, to add fractions with the same denominator, you add the numerators and place the sum over the common denominator. This leads to the following definition.

Fraction Addition

If $a, b, and\ c$ are numbers where $c \neq 0$, then $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

6) Use the definition of fraction addition in the box above to add $\frac{6}{23} + \frac{8}{23}$.

(a) Identify a, b, and c.

(b) Add the fractions.

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7) Explain in words how to add two fractions that have the same denominator.

Model Fraction Addition – Extra Practice

Use fraction circles to model each addition. Sketch your model and write the sum.

You may want to use the fraction circles on the interactive website http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

1) $\frac{1}{5}+\frac{2}{5}$

2) $\frac{1}{6}+\frac{2}{6}$

3) $\frac{3}{8}+\frac{1}{8}$

4) $\frac{4}{10}+\frac{1}{10}$

5) $\frac{3}{10}+\frac{3}{10}$

6) $\frac{5}{12}+\frac{5}{12}$

7) $\frac{5}{9}+\frac{3}{9}$

8) $\frac{3}{8}+\frac{4}{8}$

9) $\frac{4}{5}+\frac{2}{5}$

10) $\frac{4}{9}+\frac{6}{9}$

11) $\frac{5}{8}+\frac{7}{8}$

12) $\frac{7}{10}+\frac{9}{10}$

Model Fraction Subtraction

Resources Needed:

Each student needs the worksheet and a set of fractions circles.

Background:

Many students who take this course have never been comfortable with fractions. Fraction addition and subtraction are especially problematic. Students don't understand why adding or subtracting across numerators and/or denominators is incorrect. Working with concrete models helps students see why a common denominator is needed.

Directions:

- This activity may be done by students individually or in small groups. The addition and subtraction worksheets may be done at separate times.
- Give each student a set of fraction circles and the worksheet.
- Discuss the example of modeling subtraction involving pizza. You may want to illustrate this on the board.
- Then use fraction circles to model the same subtraction, $\frac{7}{12}-\frac{2}{12}$.
- Let the class proceed through the worksheet activity.
- When most of the students have completed the worksheet, bring the whole class together. Discussion at the end of this activity will help reinforce the concepts. Since the goal is to have students understand subtraction of fractions with common denominators, you may want to accept answers that are not simplified, such as $\frac{6}{8}$.
- You may want to show your students how to use the fraction circles on the website http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

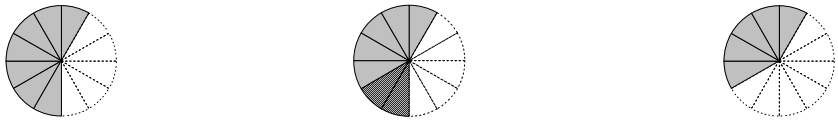
Manipulative Mathematics **Name**_____

Model Fraction Subtraction

Subtracting two fractions with common denominators works the same as addition of fractions with common denominators. Think of a pizza that was cut into twelve equal slices. Each piece is $\frac{1}{12}$ of the pizza. After dinner there are seven pieces, $\frac{7}{12}$, left in the box. If Leonardo eats 2 of the pieces, $\frac{2}{12}$, how much is left? There would be 5 pieces left, $\frac{5}{12}$. So $\frac{7}{12}-\frac{2}{12}=\frac{5}{12}$.

1) Let's use Fraction Circles to model the same example, $\frac{7}{12}-\frac{2}{12}$.

(a) Start with seven $\frac{1}{12}$ pieces. Take away two $\frac{1}{12}$ pieces.



How many twelfths do you have left?_____

(b) You have five pieces left, $\frac{5}{12}$. $\frac{7}{12}-\frac{2}{12}=$ _____

2) Use your fraction circles to model the difference $\frac{4}{5}-\frac{1}{5}$.

Start with four $\frac{1}{5}$ pieces. Take away one $\frac{1}{5}$ piece.

(a) How many fifths do you have left?_____

(b) Sketch your model here.

(c) You have _____ fifths left. $\frac{4}{5}-\frac{1}{5}=$ _____

3) Use fraction circles to model the following. Sketch a diagram to illustrate your model**.**

(a) $\frac{7}{8}-\frac{4}{8}=$ _____ (b) $\frac{5}{6}-\frac{4}{6}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

4) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{3}{4}-\frac{2}{4}=$ _____ (b) $\frac{4}{5}-\frac{2}{5}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

5) Use fraction circles to model the following. Sketch a diagram to illustrate your model.

(a) $\frac{5}{8}-\frac{2}{8}=$ _____ (b) $\frac{7}{10}-\frac{4}{10}=$ _____

(c) Look at parts (a) and (b). Explain how you got the numerator and denominator of your answers.

These examples show that to subtract the same size fraction pieces—that is, fractions with the same denominator—you just subtract the number of pieces. So, to subtract fractions with the same denominator, you subtract the numerators and place the difference over the common denominator. This leads to the following definition.

Fraction Subtraction

If $a, b, and\ c$ are numbers where $c \neq 0$, then $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$

6) Use the definition of fraction subtraction in the box above to subtract $\frac{11}{17}-\frac{5}{17}$.

(a) Identify a, b, and c.

(b) Subtract the fractions.

7) Explain in words how to subtract two fractions that have the same denominator.

Model Fraction Subtraction – Extra Practice

Use fraction circles to model each subtraction. Sketch your model and write the difference.

You may want to use the fraction circles on the interactive website http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

1) $\frac{3}{5}-\frac{1}{5}$

2) $\frac{5}{6}-\frac{1}{6}$

3) $\frac{7}{8}-\frac{1}{8}$

4) $\frac{9}{10}-\frac{1}{10}$

5) $\frac{3}{10}-\frac{3}{10}$

6) $\frac{5}{12}-\frac{5}{12}$

7) $\frac{5}{9}-\frac{3}{9}$

8) $\frac{4}{8}-\frac{3}{8}$

9) $\frac{6}{5}-\frac{2}{5}$

10) $\frac{10}{9}-\frac{4}{9}$

11) $\frac{13}{8}-\frac{5}{8}$

12) $\frac{17}{10}-\frac{7}{10}$

Model Finding the Least Common Denominator

Resources Needed:

Each student needs a worksheet and a set of fractions tiles or fraction circles.

Background:

Many students who take this course have never been comfortable with fractions. Addition and subtraction of fractions with different denominators is confusing and often meaningless. While they may remember when a common denominator is needed, they generally have no conceptual understanding of what that common denominator means. As a result, when asked to convert to equivalent fractions with an LCD they follow routine procedures without a visual image of what they are really accomplishing.

Directions:

- This activity may be done by students in small groups or as individuals. Each student should complete his or her own worksheet.
- Give each student a set of fraction tiles or fraction circles and a worksheet. Since the activity can be done using either fraction tiles or fraction circles, the worksheet refers generically to 'fraction pieces'.
- Talk through the example of adding one quarter and one dime. Most students will say they 'just know' the sum is 35 cents. Explicitly relate this to the idea of a common denominator. You might want to remind them that the coin 'quarter' is named because its value is one-quarter $\left(\frac{1}{4}\right)$ of one dollar.
- Then model exercise 1, parts (a) through (d). Make sure students understand what is meant when they are asked 'to find a common fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{3}$ exactly'. Show them how they can cover $\frac{1}{2}$ exactly with $\frac{1}{4}$ pieces but they cannot cover $\frac{1}{3}$ exactly with $\frac{1}{4}$ pieces. (Let your students know whether you prefer them to answer question 1(d) by saying 'it takes two $\frac{1}{4}$ pieces to cover $\frac{1}{3}$' or 'it cannot be done'.)
- Let the class proceed through the worksheet activities. Walk around the room, to make sure all students are on task and to answer any individual questions that arise.
- Discussion at the end of this activity will help reinforce the concepts. The worksheet does not use the phrase 'equivalent fraction', but students do convert fractions to equivalent fractions with the LCD, so you may want to point that out.
- You may want to show your students how to use the fraction tiles on the website http://www.mathsisfun.com/numbers/fraction-number-line.html or the fraction circles at http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html.

Model Finding the Least Common Denominator

Let's look at coins again. Can you add one quarter and one dime? Well, you could say there are two coins, but that's not very useful. To find the total value of one quarter plus one dime, you change them to the same kind of unit – cents. One quarter equals 25 cents and one dime equals 10 cents, so the sum is 35 cents.


One quarter + one dime

25¢ + 10¢

35¢

25¢ + 10¢

$\frac{25}{100} + \frac{10}{100}$

$\frac{35}{100}$

Similarly, when you add fractions with different denominators you have to convert them to equivalent fractions with a common denominator. With the coins, when we converted to cents, the denominator was 100. 25 cents is $\frac{25}{100}$ and 10 cents is $\frac{10}{100}$ and so we added $\frac{25}{100}+\frac{10}{100}$ to get $\frac{35}{100}$, which is 35 cents.

- Use fraction pieces to find the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$. Take out your set of fraction pieces and place $\frac{1}{2}$ and $\frac{1}{3}$ on your workspace. You need to find a common fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{3}$ **exactly**.

1) Try the $\frac{1}{4}$ pieces.

(a) Can you cover the $\frac{1}{2}$ piece exactly with $\frac{1}{4}$ pieces? _____

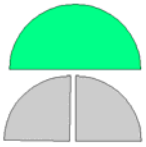
(b) How many $\frac{1}{4}$ pieces cover the $\frac{1}{2}$ piece? _____

(c) Can you cover the $\frac{1}{3}$ piece exactly with $\frac{1}{4}$ pieces? _____

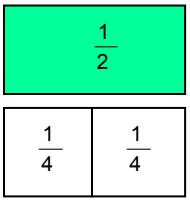
(d) How many $\frac{1}{4}$ pieces cover the $\frac{1}{3}$ piece?_____

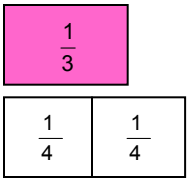
(e) Sketch your results here.

fraction circles



fraction tiles





2) Try the $\frac{1}{5}$ pieces.

(a) Can you cover the $\frac{1}{2}$ piece exactly with $\frac{1}{5}$ pieces? _____

(b) How many $\frac{1}{5}$ pieces cover the $\frac{1}{2}$ piece?___

(c) Can you cover the $\frac{1}{3}$ piece exactly with $\frac{1}{5}$ pieces?_____.

(d) How many $\frac{1}{5}$ pieces cover the $\frac{1}{3}$ piece?___

(e) Sketch your results here.

3) Try the $\frac{1}{6}$ pieces.

(a) Can you cover the $\frac{1}{2}$ piece exactly with $\frac{1}{6}$ pieces? _____

(b) How many $\frac{1}{6}$ pieces cover the $\frac{1}{2}$ piece? ___

(c) Can you cover the $\frac{1}{3}$ piece exactly with $\frac{1}{6}$ pieces? _____.

(d) How many $\frac{1}{6}$ pieces cover the $\frac{1}{3}$ piece? ___

(e) Sketch your results here.

4) You have shown that:

(a) 3 of the $\frac{1}{6}$ pieces exactly cover the $\frac{1}{2}$ piece. $\frac{1}{2} = \frac{\quad}{6}$

(b) 2 of the $\frac{1}{6}$ pieces exactly cover the $\frac{1}{3}$ piece. $\frac{1}{3} = \frac{\quad}{6}$

The smallest denominator of a fraction piece that can be used to cover both fractions exactly is the **least common denominator** (LCD) of the two fractions. The smallest denominator of a fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{3}$ is 6. So, you have found that the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

- Use fraction pieces to find the least common denominator of $\frac{1}{4}$ and $\frac{1}{6}$. Place $\frac{1}{4}$ and $\frac{1}{6}$ on your workspace. Find a common fraction piece that can be used to cover both $\frac{1}{4}$ and $\frac{1}{6}$ exactly.

5) Sketch your results here.

6) You have shown that:

(a) _____ of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{4}$ piece. $\frac{1}{4} = \frac{\quad}{\quad}$

(b) _____of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{6}$ piece. $\frac{1}{6} = \frac{\quad}{\quad}$

(c) Both fractions can be written with denominator_____, so _____ is their common denominator.

- Use fraction pieces to find the least common denominator of $\frac{1}{4}$ and $\frac{1}{3}$. Find a common fraction piece that can be used to cover both $\frac{1}{4}$ and $\frac{1}{3}$ exactly.

7) Sketch your results here.

8) You have shown that:

(a) _____ of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{4}$ piece . $\frac{1}{4} = \frac{\quad}{\quad}$

(b) _____of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{3}$ piece. $\frac{1}{3} = \frac{\quad}{\quad}$

(c) Both fractions can be written with denominator_____, so ____ is their common denominator.

- Use fraction pieces to find the least common denominator of $\frac{1}{2}$ and $\frac{1}{5}$. Find a common fraction piece that can be used to cover both $\frac{1}{2}$ and $\frac{1}{5}$ exactly.

9) Sketch your results here.

10) You have shown that:

(a) _____ of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{2}$ piece. $\frac{1}{2} = \frac{\quad}{\quad}$

(b) _____of the $\frac{1}{\square}$ pieces exactly cover the $\frac{1}{5}$ piece. $\frac{1}{5} = \frac{\quad}{\quad}$

(c) Both fractions can be written with denominator_____, so ____ is their common denominator.

Model Finding the Least Common Denominator – Extra Practice

Use fraction tiles or fraction circles to find the least common denominator (LCD) of each pair of fractions, and to re-write each fraction with the LCD. Sketch your model.

You may want to use the fraction tiles on the interactive website http://www.mathsisfun.com/numbers/fraction-number-line.html or the fraction circles at http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&hidepanel=true&from=topic_t_1.html to work these exercises.

1) $\frac{1}{3}$ and $\frac{1}{6}$

(a) LCD = _____

(b) $\frac{1}{3}$ = _____

(c) $\frac{1}{6}$ = _____

(d) sketch your model.

2) $\frac{1}{2}$ and $\frac{1}{8}$

(a) LCD = _____

(b) $\frac{1}{2}$ = _____

(c) $\frac{1}{8}$ = _____

(d) sketch your model.

3) $\frac{2}{3}$ and $\frac{1}{12}$

(a) LCD = _____

(b) $\frac{2}{3}$ = _____

(c) $\frac{1}{12}$ = _____

(d) sketch your model.

4) $\frac{3}{4}$ and $\frac{1}{12}$

(a) LCD = _____

(b) $\frac{3}{4}$ = _____

(c) $\frac{1}{12}$ = _____

(d) sketch your model.

5) $\frac{3}{8}$ and $\frac{3}{4}$

(a) LCD = _____

(b) $\frac{3}{8}$ = _____

(c) $\frac{3}{4}$ = _____

(d) sketch your model.

6) $\frac{5}{6}$ and $\frac{2}{3}$

(a) LCD = _____

(b) $\frac{5}{6}$ = _____

(c) $\frac{1}{2}$ = _____

(d) sketch your model.

7) $\frac{1}{2}$ and $\frac{2}{5}$

(a) LCD = _____

(b) $\frac{1}{2}$ = _____

(c) $\frac{2}{5}$ = _____

(d) sketch your model.

8) $\frac{1}{3}$ and $\frac{3}{4}$

(a) LCD = _____

(b) $\frac{1}{3}$ = _____

(c) $\frac{3}{4}$ = _____

(d) sketch your model.

9) $\frac{5}{12}$ and $\frac{2}{3}$

(a) LCD = _____

(b) $\frac{5}{12}$ = _____

(c) $\frac{2}{3}$ = _____

(d) sketch your model.

10) $\frac{3}{4}$ and $\frac{5}{6}$

(a) LCD = _____

(b) $\frac{3}{4}$ = _____

(c) $\frac{5}{6}$ = _____

(d) sketch your model.