

Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Multiples and Factors

Model Multiplication and Factoring
Square Numbers

Manipulatives used:

Square 'color' tiles
Counters

Manipulative Mathematics

Model Multiplication and Factoring

Instructor Page

Resources Needed:

Each student needs a set of 25 square tiles (often called 'color tiles') and a sheet of graph paper.

Background Information:

Many students have a limited visual understanding of multiplication. This activity is designed to give them a visual representation of multiplication as they build rectangular arrays. These arrays and activities will help them understand factors, the commutative property of multiplication, and prime numbers.

Directions:

- This exercise usually works best with students working in pairs or groups. Give each student a set of 25 color tiles.
- Work exercise 1 together with the class. Show your students how you can make one rectangle using 12 tiles, and then ask them to arrange 12 of their tiles into another rectangle. Ask the students to describe their rectangles, and draw and label each one. Continue asking students for more rectangles until all 3 unique combinations (1x12, 2x6, 3x4) are displayed. As you label each rectangle with its dimensions (ie, 2x6), explain that a 2x6 and a 6x2 (etc.) rectangle are really the same. You may want to rotate or superimpose a model to show this. Have the students color a representation of the three possible rectangles on their graph paper and label each '12'.
- Then have students proceed through the worksheet. They will find all possible rectangles using exactly 1 tile, 2 tiles, 3 tiles, ..., 25 tiles. Students will draw their rectangles on their graph paper and label the rectangles with their dimensions.
- These rectangles and the summary chart can lead into rich discussions about factors, primes, multiples, square numbers, etc. Allow time for discussion to develop in the groups and then bring the class together for further discussion.
- You may want to demonstrate exercise 1 by using the square pattern blocks at http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_3.html?open=activities&from=topic_t_3.html.

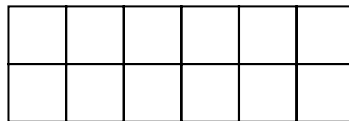
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Model Multiplication and Factoring

Name _____

- 1) Take twelve tiles and form a rectangle.
 - (a) How many rows does your rectangle have? _____
 - (b) How many columns? _____
 - (c) Each rectangle can be called a $\frac{\text{number of rows}}{\text{number of rows}} \times \frac{\text{number of columns}}{\text{number of columns}}$ rectangle.

The number of rows and the number of columns are called the **dimensions** of the rectangle.



____ rows by ____ columns

____ \times ____ rectangle

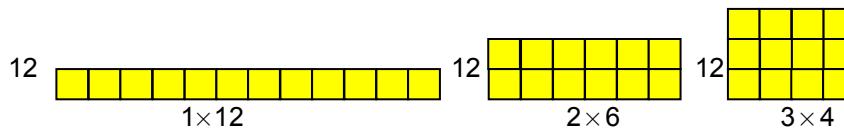
- (d) How many tiles were used to make this rectangle? _____
- (e) What is the product of $2 \cdot 6$? _____
- (f) Form a 6×2 rectangle. Draw it here:

- (g) What do you notice about the 2×6 and the 6×2 rectangles?

A 2×6 rectangle and a 6×2 rectangle are **equivalent**. This means you could rotate the 2×6 rectangle and it would look exactly the same as the 6×2 rectangle.

- 2) Now, create all possible rectangles using 1 tile, 2 tiles, 3 tiles, ..., 25 tiles.
 - (a) Copy each rectangle onto graph paper.
 - (b) Label each rectangle with the total number of tiles used to form it.
 - (c) Under the rectangle write its dimensions: $\frac{\text{number of rows}}{\text{number of rows}} \times \frac{\text{number of columns}}{\text{number of columns}}$.

For example, your graph paper would show 3 rectangles for 12 tiles:



- (d) Summarize your results in the chart below.

Number of tiles	Dimensions of the rectangles formed
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	1×12 , 2×6 , 3×4
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	

Use your chart to answer the following questions.

3) Look for all the rectangles in your chart that have 2 rows.

(a) List the dimensions of all the rectangles that have 2 rows.

2×1 , 2×2 , _____, _____, _____, _____, _____, _____, _____, _____, _____

(b) Now list the total number of tiles you used to form each rectangle you listed in 5.

2, 4, _____, _____, _____, _____, _____, _____, _____, _____, _____

These numbers are called the **multiples** of 2.

Multiple

A number is a multiple of n if it is the product of a counting number and n .

4) How can you use your rectangle chart to find the multiples of 3?

5) List the multiples of three: 3, ____, ____, ____, ____, ____, ____, ____.

6) List the multiples of four: 4, ____, ____, ____, ____, ____.

7) List the multiples of five: 5, ____, ____, ____, ____.

Notice that with 12 tiles, we could form 3 different rectangles, 1×12 , 2×6 , and 3×4 . The numbers 1, 2, 3, 4, 6, and 12 are **factors** of 12, because $1 \cdot 12 = 12$, $2 \cdot 6 = 12$, and $3 \cdot 4 = 12$.

Factors

If $a \cdot b = m$, then a and b are factors of m .

8) List all the factors of 15: ____, ____, ____, ____.

9) Which number from 1 to 25 has the most factors? _____

10) Which number of tiles can be used to make the most rectangles? _____

11) Explain why some numbers can be used to make more rectangles than other numbers.

12) List the numbers for which you could only form one rectangle.

These numbers are called **primes**. A prime number has only two factors, 1 and itself.

Prime

A prime number is a counting number greater than 1, whose only factors are 1 and itself.

13) List all the primes between 2 and 25.

14) What other number relationships do you notice in your rectangle chart?

Model Multiplication and Factoring– Extra Practice

List the first ten multiples of the following numbers.

1) 6: _____, _____, _____, _____, _____, _____, _____, _____, _____, _____

2) 7: _____, _____, _____, _____, _____, _____, _____, _____, _____, _____

3) 8: _____, _____, _____, _____, _____, _____, _____, _____, _____, _____

4) 9: _____, _____, _____, _____, _____, _____, _____, _____, _____, _____

5) 12: _____, _____, _____, _____, _____, _____, _____, _____, _____, _____

The number 16 can be factored $1 \cdot 16$, $2 \cdot 8$, and $4 \cdot 4$, so all the factors of 16 are 1, 2, 4, 8, and 16. Find all the factors of each of the following numbers.

6) 24 _____

7) 30 _____

8) 42 _____

9) 63 _____

10) 135 _____

Manipulative Mathematics

Square Numbers

Instructor Page

Resources Needed:

Each student needs a worksheet and about 50 counters.

Background Information:

This activity is designed to give students a visual representation of the square of a number. While this exercise may seem almost trivial to a teacher, it can be difficult for some students. They may never have realized how the word 'square' in 3^2 relates to a shape. Usually, once students begin to see the concept of square, they make the connection between the numbers and the shapes. The rewards of this activity validate the time spent. Students are able to identify perfect squares more easily, which is a great help as they begin factoring.


Directions:


- This activity is best done by students working individually at the start. Students will need a partner to do Exercise 4.
- Give each student a worksheet and about 50 counters. (50 counters will allow them to model square numbers through $7^2 = 49$.)
- Model Exercise 1a for the class. Show students the concept of the square number 4 using counters.
- Have the students continue through the worksheet. When they get to the Exercise 4, they will need to work with a partner.
- When most students have completed the activity, bring the class together for discussion. You may want to review the definition of square numbers shown above Exercise 5. If you have students in your class who speak languages other than English, you may want to ask them how to say, for example, 7^2 and the shape 'square' in their language. Chances are they are both the same word!
- You may want to demonstrate exercise 1 by using the square pattern blocks at http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_3.html?open=activities&from=topic_t_3.html.

Manipulative Mathematics

Square Numbers

Name _____

- 1) Put about 50 color counters on your workspace. We will use the counters to make squares.
 - (a) For example,  is a square made from ____ counters. It has ____ counters on each side.
 - (b) Make as many squares as you can with your counters. Draw a picture of each square that you create and record your results in the table below:

Picture of square							
Total number of counters in the square		4					
Number of counters on each side		2					

- 2) Can you make a square with exactly 6 counters? ____ Why or why not?

- 3) Imagine if you had 100 counters
 - (a) Could you make a square with exactly 100 counters? ____
 - (b) Why or why not?

 - (c) How many counters would be on each side of a square made with 100 counters? ____

- 4) Work with a partner and put all your counters together.
 - (a) Create a square that uses more than 50 counters. Draw a sketch of your square.

(b) Create all the squares you can using 50 to 100 counters. Sketch your squares here.

When a number n is multiplied by itself, we write it n^2 and read it 'n squared'. For example,

8^2 is read '8 squared'.

64 is called 'the square of 8'.

Similarly, 121 is the square of 11, because 11^2 is 121.

Square of a number

If $n^2 = m$, then m is the square of n .

5) Complete this table to show the squares of the counting numbers 1 through 15.


n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n^2								64			121				

The squares of the counting numbers are called **perfect squares**, so the second row of the table shows the first fifteen perfect squares.

6) List the total number of counters you used for each square you made in Exercise 1(b).

7) Do you see a similarity between the table you filled in for Exercise 1 with the pictures of squares and the table you made in Exercise 5 with the squares of the counting numbers 1 through 15? ____

(a) Describe how the two tables are alike.

(b) Why do we use the word 'square' for both the symbol in 3^2 and the shape  ?

Manipulative Mathematics
Square Numbers – Extra Practice

Name _____

Identify whether or not each number is a perfect square. If it is a perfect square, write it as the square of a counting number.

Number	Not a perfect square	Yes – perfect square
1) 36	_____	_____ = (_____) ²
2) 50	_____	_____ = (_____) ²
3) 140	_____	_____ = (_____) ²
4) 196	_____	_____ = (_____) ²
5) 221	_____	_____ = (_____) ²
6) 289	_____	_____ = (_____) ²
7) 364	_____	_____ = (_____) ²
8) 625	_____	_____ = (_____) ²
9) 784	_____	_____ = (_____) ²
10) 961	_____	_____ = (_____) ²