Manipulative Mathematics
Using Manipulatives to Promote Understanding of Math Concepts

Signed Numbers
Addition of Signed Numbers
Subtraction of Signed Numbers

Manipulatives used:
Two color counters

Manipulative Mathematics   **Instructor Page**
Addition of Signed Numbers

Resources Needed:

Each student needs about 30 two-color counters. If you have no supply money to purchase counters, buy a bag of red beans and a bag of white beans at a grocery store. Beans are inexpensive and easy to replace, so students can take them home and use them to do the homework. Each student needs about 15 red beans and 15 white beans.

Background Information:

Most students understand addition and subtraction of positive numbers since they have worked with them for many years, but when negative numbers are introduced students encounter difficulties. Students tend to merely want a 'rule' to follow just to get the answer. By using two-color counters, students have a concrete model of the abstract concepts of signed numbers, and develop the 'rules' themselves. Furthermore, the model for subtraction of signed numbers agrees with the 'take away' idea most students used when they first learned subtraction as children.

Directions:

- This activity introduces addition of signed numbers.

- Give each student about 30 two-color counters (or about 15 red beans and 15 white beans).

- State that the red side of a counter (or one red bean) represents one negative unit and the other side (one white bean) represents one positive unit. Using red for negative is consistent with the 'red ink' used in accounting.

- Explain that one positive and one negative together make a 'neutral pair'. The value of a neutral pair is zero.

- Demonstrate worksheet exercises 1 through 4, having the students model with their counters, too. When you get a neutral pair, physically remove it from the workspace.

- After working exercises 1 through 4 together, you may want your students to model $6+4$, $6+(-4)$, $-6+4$, $-6+(-4)$ on their own and then reinforce the correct methods by modeling them yourself for the class.

- Let students work in groups of 2 or 3 on the rest of the worksheet. Make sure everyone is actually modeling the sums instead of just writing the answers. It will be important that they can model sums before attempting subtraction.

- When most groups are finished, bring the class back together.   Discuss their answers to exercises 17 and 18.

- Ideally, addition would be introduced in class one day and the students would do addition exercises for homework. Then subtraction would be introduced at the next class meeting.

- Students can get additional practice using two color counters to add signed numbers online at the National Library of Virtual Manipulatives website http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html?from=topic_t_1.html.

Manipulative Mathematics   **Name**_____
Addition of Signed Numbers

Team
Members_____

We are going to model signed numbers with two-color counters. One white counter, ○, will represent one positive unit. One red counter, ●, will represent one negative unit.

When we have one positive and one negative together, ○● we call it a 'neutral pair'. The value of a neutral pair is zero.

1) We'll start by modeling $5 + 3$, the sum of 5 and 3.

   (a) Start with 5 positives.   ○ ○ ○ ○ ○

   (b) Add 3 positives. Put counters of the same color   ○ ○ ○ ○ ○ ○ ○ ○
       in the same row.

   (c) How many counters are there?   _____ positives

$$5 + 3 = 8$$

2) Now we'll model $-5 + (-3)$, the sum of negative 5 and negative 3.

   (a) Start with 5 negatives.   ● ● ● ● ●

   (b) Add 3 negatives.   ● ● ● ● ● ● ● ●

   (c) How many counters are there?   _____ negatives

$$-5 + (-3) = -8$$

3) What about adding numbers with different signs? Let's model $-5 + 3$, the sum of negative 5 and 3.

   (a) Start with 5 negatives.   ● ● ● ● ●

   (b) Add 3 positives. Since they are a different color,   ● ● ● ● ●
       line them up under the red counters.   ○ ○ ○

   (c) Are there any neutral pairs? _____   ● ● ● ● ●
       Remove the neutral pairs.   ○ ○ ○

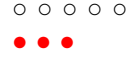   (d) How many are left?   ● ● _____ negatives

$$-5 + 3 = -2$$

4) The fourth case is the sum of a positive and a negative. We'll model $5 + (-3)$, the sum of 5 and negative 3.
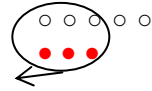
   (a) Start with 5 positives.   ○ ○ ○ ○ ○

(b) Add 3 negatives.   ○ ○ ○ ○ ○
                                                     ● ● ●

(c) Remove the neutral pairs.   ○ ○ ○ ○ ○
                                                     ● ● ●

(d) How many are left?   ○ ○ 2 _____

$$5+(-3)=2$$

Use your counters to model each sum. Draw a sketch of your model.

5) $4+2$ 6) $-5+(-5)$ 7) $-1+4$ 8) $2+(-4)$

9) $8+(-4)$ 10) $7+(-3)$ 11) $-2+(-3)$ 12) $-5+7$

13) $-2+(-1)$ 14) $-3+3$ 15) $7+(-2)$ 16) $-4+2$

17) Do you notice a pattern? Explain in words how to add:
    (a) $-8+(-10)$

    (b) $25+(-5)$

18) Without using counters, try to find these sums.
    (a) $35+29$ (b) $-57+(-43)$

    (c) $78+(-74)$ (d) $-64+31$

Manipulative Mathematics   **Name**_____

Addition of Signed Numbers – Extra Practice

Use two-color counters to model each addition.

You can find virtual counters on the website
http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html?from=topic_t_1.html. If you use the
website, click on 'User' at the bottom of the workspace so that you can enter the numbers in each
exercise.

Sketch the model for each addition and find the sum.

1) $5 + 2$

2) $-5 + (-2)$

3) $-5 + 2$

4) $5 + (-2)$

5) $6 + (-6)$

6) $3 + (-1)$

7) $-4 + (-5)$

8) $-6 + 8$

9) $-3 + (-7)$

10) $-2 + 2$

11) $4 + (-8)$

12) $-4 + 9$

Subtraction of Signed Numbers

Resources Needed:

> Each student needs about 30 two-color counters. If you have no supply money to purchase counters, buy a bag of red beans and a bag of white beans at a grocery store. Beans are inexpensive and easy to replace, so students can take them home and use them to do the homework. Each student needs about 15 red beans and 15 white beans.

Background Information:

> Most students understand addition and subtraction of positive numbers since they have worked with them for many years, but when negative numbers are introduced students encounter difficulties. Students tend to merely want a 'rule' to follow just to get the answer. By using two-color counters, students have a concrete model of the abstract concepts of signed numbers, and develop the 'rules' themselves. Furthermore, the model for subtraction of signed numbers agrees with the 'take away' idea most students used when they first learned subtraction as children.

Directions:

- This activity introduces subtraction of signed numbers. Students will discover that $a - b = a + (-b)$. Ideally, students would have been introduced to addition of signed numbers at the previous class meeting and have done homework to practice.

- Give each student about 30 two-color counters (or about 15 red beans and 15 white beans).

- Remind students that the red side of a counter (or one red bean) represents one negative unit and the other side (one white bean) represents one positive unit. Review the idea of a 'neutral pair' – one positive and one negative together. The value of a neutral pair is zero.

- Demonstrate worksheet exercises 1 through 4, having the students model with their counters, too. When you get a neutral pair, physically remove it from the workspace.

- After working exercises 1 through 4 together, you may want your students to model $6 - 4$, $-6 - (-4)$, $-6 - 4$, $6 - (-4)$ on their own and then reinforce the correct methods by modeling them yourself for the class.

- Let students work in groups of 2 or 3 on the rest of the worksheet. Make sure everyone is actually modeling the differences instead of just writing the answers. A quick look at the answers to exercises 13 and 14 will let you know if the students understand the process.

- When most groups are finished, bring the class back together.   Discuss their answers to exercises 17 and 18.

- Students can get additional practice using two color counters to subtract signed numbers online at the National Library of Virtual Manipulatives website http://nlvm.usu.edu/en/nav/frames_asid_162_g_3_t_1.html?from=topic_t_1.html.

Manipulative Mathematics   **Name**_____

Subtraction of Signed Numbers

Team Members_____

We are going to model signed numbers with two-color counters. One white counter, ○, will represent one positive unit. One red counter, ●, will represent one negative unit.

When we have one positive and one negative together, ○ ● we call it a 'neutral pair'. The value of a neutral pair is zero.

1) We'll start by modeling $5 - 3$, the difference of 5 and 3.
   (a) Start with 5 positives.   ○ ○ ○ ○ ○

   (b) Take away 3 positives.   ○ ○ ○ ○ ○

   (c) How many counters are left?   _____ positives

                                                                         $5 - 3 = 2$

2) Now we'll model $-5 - (-3)$, the difference of negative 5 and negative 3.
   (a) Start with 5 negatives.   ● ● ● ● ●

   (b) Take away 3 negatives.   ● ● ● ● ●

   (c) How many counters are left?   ● ● _____ negatives

                                                                 $-5 - (-3) = -2$

3) What about subtracting numbers with different signs? Let's model $-5 - 3$, the difference of negative 5 and 3.
   (a) Start with 5 negatives.   ● ● ● ● ●

   (b) We want to take away 3 positives. Do we have
       any positives to take away?   _____

   (c) We can add 3 neutral pairs to get the 3 positives.   ● ● ● ● ● ● ● ●
                                                                              ○ ○ ○

   (d) Now take away 3 positives.   ● ● ● ● ● ● ● ●
                                                                              ○ ○ ○

   (e) How many counters are left?   ● ● ● ● ● ● ● ●

                                                                 8 _____

                                                                 $-5 - 3 = -8$

4) The fourth case is the sum of a positive and a negative. We'll model $5-(-3)$, the difference of 5 and negative 3.

(a) Start with 5 positives. ○ ○ ○ ○ ○

(b) We want to take away 3 negatives. Do we have any negatives to take away? _____

(c) But we can add 3 neutral pairs to get the 3 negatives. ○ ○ ○ ○ ○   ○ ○ ○
                                                                    ● ● ●

(d) Take away 3 negatives. ○ ○ ○ ○ ○   ○ ○ ○
                                       ● ● ●

(e) How many counters are left? ○ ○ ○ ○ ○   ○ ○ ○

8 _____

$$5-(-3)=8$$

Use your counters to model each difference. Draw a sketch of your model.

5) $7-2$ 6) $6-(-4)$ 7) $-1-4$ 8) $-3-(-2)$

9) $3-(-4)$ 10) $-5-(-1)$ 11) $8-6$ 12) $-7-3$

13) $-4-(-4)$ 14) $-3-3$ 15) $1-5$ 16) $-2-(-6)$

17) Do you notice a pattern? Explain in words how to subtract:

(a) $-8-(-2)$

(b) $-10-5$

18) Without using counters, try to find these differences.

(a) $35-29$ (b) $-57-(-43)$

(c) $78-(-74)$ (d) $-64-31$

Manipulative Mathematics   **Name**_____

Subtraction of Signed Numbers – Extra Practice

Use two-color counters to model each subtraction.

You can find virtual counters on the website
http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html?from=topic_t_1.html. If you use the website, click on 'User' at the bottom of the workspace so that you can enter the numbers in each exercise.

Sketch the model for each subtraction and find the difference.

1) $7 - 2$ 2) $7 - (-2)$ 3) $-7 - 2$ 4) $-7 - (-2)$

5) $6 - (-5)$ 6) $-4 - (-1)$ 7) $-8 - 8$ 8) $9 - 5$

9) $-3 - (-3)$ 10) $5 - 4$ 11) $-2 - (-6)$ 12) $4 - 10$