Manipulative Mathematics

Using Manipulatives to Promote Understanding of Math Concepts

Slopes

Exploring Slopes of Lines
Slope of Line Between Two Points

Manipulatives used:
Geoboards

Manipulative Mathematics **Instructor Page**

Exploring Slopes of Lines

Resources Needed:

Each student needs a worksheet, a geoboard, and 3 or more rubber bands. If geoboards are not available, you can photocopy the last page of this packet.

Background Information:

Slope is a concept for which doing a simple concrete exercise may make a substantial difference in student understanding. This activity introduces students to the $\frac{\text{rise}}{\text{run}}$ definition of slope. Students will model a line on a geoboard with a rubber band, then form a right triangle and count the rise and the run to determine the slope of the line. For students new to the concept of slope, it helps to consistently work from the left of the geoboard to the right and find the rise first and then the run, in the same order they will put the numbers into the formula $m = \frac{\text{rise}}{\text{run}}$. Also, when working from left to right, the run is always positive, while the rise is positive or negative, depending on whether the rubber band is stretched up or down. 'Reading' a graph from left to right will also be consistent with the concepts of increasing or decreasing functions, which students encounter in higher level math classes. By completing this activity as well as "Slopes of Lines Between Two Points" students gain comprehension of this abstract topic.

Directions:

- Pass out the geoboards and rubber bands. Ideally, each student should have a geoboard, but the activity can be done with students working in pairs.
- Work through Exercise 1 with the whole class. Begin by demonstrating how to represent a line on the geoboard with a rubber band, then stretch the rubber band up and to the right to form a right triangle as shown in 1(b). Emphasize that the sides of the right triangle are vertical and horizontal lines. Discuss the definition of slope, then count the rise and the run in your triangle—make sure students watch you count the spaces, not the pegs. In the figure for 1(b) the rise is 6 and the run is 5.
- Next, model a line with negative slope on your geoboard. Show students how the rise is negative, but the run, still counting left to right, is positive. (Notice that in Exercise 5(b) we put the negative sign in the numerator.)
- Then let the students proceed through the worksheet. Whether they are working individually or with a partner, encourage them to compare their geoboard lines with those of their neighbors to promote dialog.
- As students work, you may want to spot check their answers to Exercises 2, 3, and 4, which are open-ended. A common error students make is to count the pegs, instead of the spaces.
- Students can get additional practice using virtual geoboards to model lines and triangles and explore slopes at the National Library of Virtual Manipulatives website http://nlvm.usu.edu/en/nav/frames_asid_279_g_4_t_3.html?open=activities&hidepanel=true&from=topic_t_3.html.

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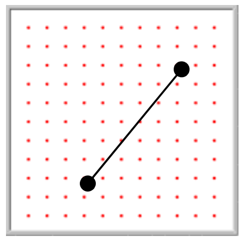
Exploring Slopes of Lines

The concept of slope has many applications in the real world. The pitch of a roof, grade of a highway, ramp for a wheelchair are some places you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

We will use geoboards to explore the concept of slope. Using rubber bands to represent lines and the pegs of the geoboards to represent points, we have a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, you'll discover how to find the slope of a line.

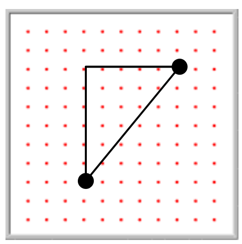
1) Let's work together to see how to use a geoboard to find the slope of a line.

(a) Take your geoboard and a rubber band. Stretch the rubber band between two pegs like this:



Doesn't it look like a line?

(b) Now stretch the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle, like this:



Be sure to make a 90º angle around the third peg, so one of the two newly formed lines is vertical and the other side is horizontal. You have made a right triangle!

To find the slope of the line count the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the **rise** and the horizontal distance is called the **run**.

> **Slope**
>
> The **slope** of a line is $m = \dfrac{\text{rise}}{\text{run}}$
>
> **rise** measures the vertical change ↕
>
> **run** measures the horizontal change ↔

(c) On your geoboard, what is the rise? ______________

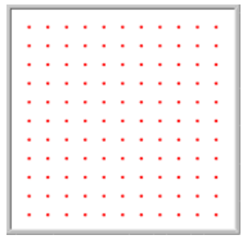
(d) What is the run? ______________

(e) What is the **slope** of the line on your geoboard?

$$m = \frac{\text{rise}}{\text{run}}$$

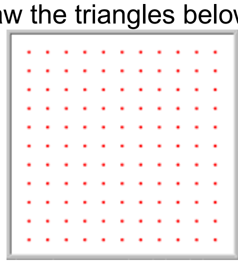
$$m = \frac{\square}{\square}$$

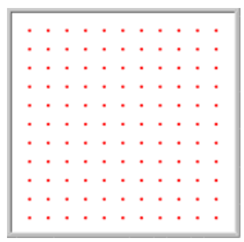
2) Make another line on your geoboard, and form its right triangle. Draw a picture of your geoboard here:

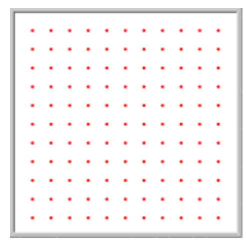


(a) What is the rise? _____ (b) What is the run? ______ (c) What is the slope? ______

3) Make 3 more lines on your geoboard, form the right triangle for each, and count their slopes. Draw the triangles below.





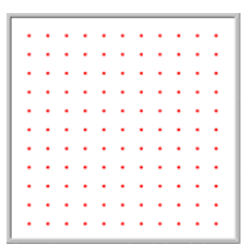


(a) Slope = _________ (b) Slope = ________ (c) Slope = _________

4) If the left endpoint of a line is higher than the right endpoint, you have to stretch the rubber band down to make the right triangle. When this happens the rise will be negative because you count down from your starting peg.

(a) Do any of your lines in exercise 3 have negative slope? ___________

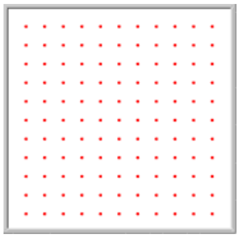
(b) Draw a line with negative slope here and calculate its slope:



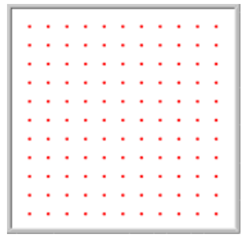
Slope = _________

5) Use a rubber band on your geoboard to make a line with each given slope and draw a picture of it.

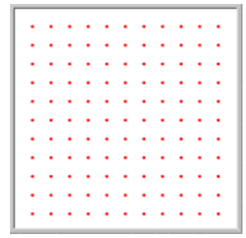
(a) Slope = $\frac{1}{3}$



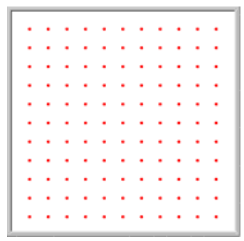
(b) Slope = $\frac{-3}{4}$



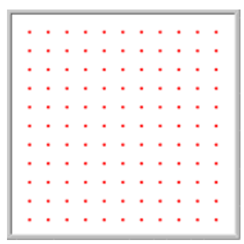
(c) Slope = 2 (hint: $2 = \frac{?}{?}$)



6) Make a horizontal line on your geoboard and draw it here. What is the slope of the horizontal line?



7) Make a vertical line on your geoboard and draw it here. What is the slope of the vertical line?



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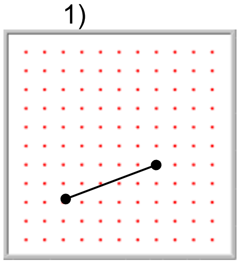
Exploring Slopes of Lines- Extra Practice

Sketch the rise and the run for the line modeled on each geoboard, then calculate the slope of the line.

You may want to use the virtual geoboard online at http://nlvm.usu.edu/en/nav/frames_asid_279_g_4_t_3.html?open=activities&hidepanel=true&from=topic_t_3.html.

1)

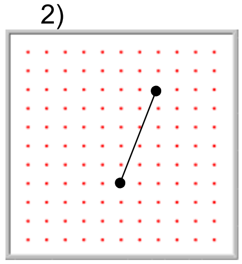


(a) rise = ____

(b) run = ____

(c) slope = ____

2)

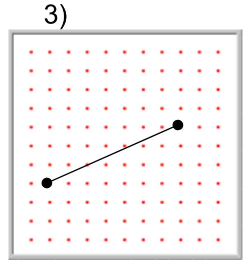


(a) rise = ____

(b) run = ____

(c) slope = ____

3)

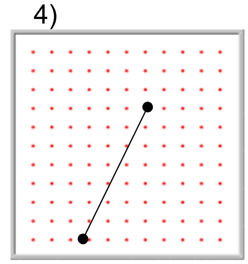


(a) rise = ____

(b) run = ____

(c) slope = ____

4)

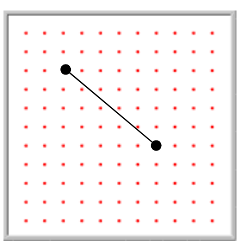


(a) rise = ____

(b) run = ____

(c) slope = ____

5)

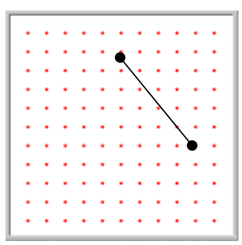


(a) rise = ____

(b) run = ____

(c) slope = ____

6)

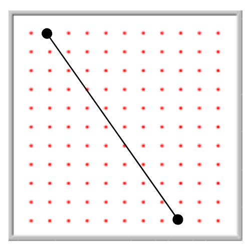


(a) rise = ____

(b) run = ____

(c) slope = ____

7)

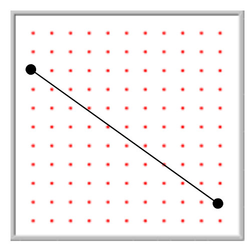


(a) rise = ____

(b) run = ____

(c) slope = ____

8)



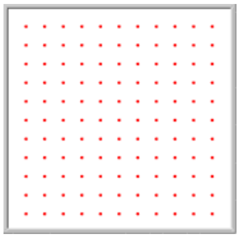
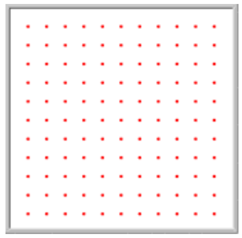
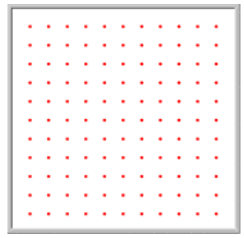
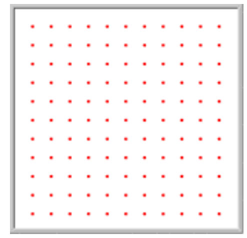
(a) rise = ____

(b) run = ____

(c) slope = ____

Draw a line with the given slope.

9) slope = $\frac{3}{10}$ 10) slope = $\frac{8}{5}$ 11) slope = $\frac{-1}{6}$ 12) slope = $\frac{-7}{4}$

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Slope of Lines Between Two Points

Resources Needed:

Each student needs a worksheet, a geoboard, and 3 or more rubber bands. If geoboards are not available, you can photocopy the last page of this packet.

Background Information:

Slope is a concept for which doing a simple concrete exercise may make a substantial difference in student understanding. In this activity, students will model a small coordinate system on a geoboard. They will locate pairs of points on the coordinate system, connect them with a rubber band to model a line, and then count the rise and run to calculate the slope.

Directions:

- Working in pairs is best to promote student dialogue, but, if possible, each student should have his or her own geoboard.
- This activity should be used following "Exploring Slopes of Lines". (The two activities may be done during the same class or on separate days.)
- After discussing "Exploring Slopes of Lines" with the class and assessing their understanding, demonstrate how to create a coordinate grid on the geoboard by stretching two rubber bands to form the x - axis and the y - axis. Spend a few minutes having students locate basic points.
- Demonstrate one example of finding the slope of a line between two points. You may wish to start by modeling a line segment with a rubber band, then identifying the coordinates of its endpoints, and then counting the rise and the run.
- Have the class do the worksheet. You may need to prompt students to "count backwards" in order to do Exercise 9.
- Notice that this activity uses the definition of slope $m = \frac{rise}{run}$. If you think your students are ready for it, you may wish to introduce the $\frac{y_2 - y_1}{x_2 - x_1}$ formula after the class has completed the worksheet.
- The National Library of Virtual Manipulatives has an online geoboard with axes at http://nlvm.usu.edu/en/nav/frames_asid_303_g_4_t_3.html?open=activities&hidepanel=true&from=topic_t_3.html.

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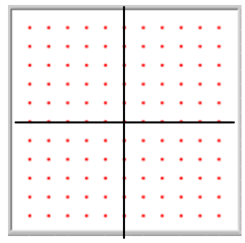
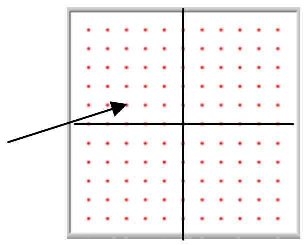
Slope of Lines Between Two Points

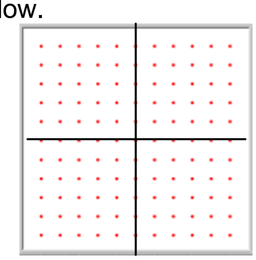
1) Start with a geoboard and 2 rubber bands. Stretch one rubber band around the middle row of pegs horizontally and the other rubber band around the middle row of pegs vertically to model the x - axis and the y - axis, like this:



You now have a small coordinate system, with $-5 \le x \le 5$ and $-5 \le y \le 5$. Each of the pegs on the geoboard represents a point on the graph. For example, the point $(-3, 1)$ is located at the arrow.



2) On your geoboard, make a line between the points $(-3, 1)$ and $(4, 3)$.
   (a) Sketch it on the geoboard below.



   (b) To find the rise and the run, stretch the rubber band into a right triangle, with one side vertical and the other horizontal. Draw your triangle on the geoboard above.

   (c) What is the rise? ____________________

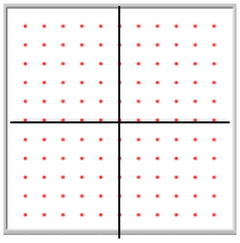
   (d) What is the run? ____________________

   (e) The slope is $m = \dfrac{rise}{run}$.

$$m = \frac{\square}{\square}$$

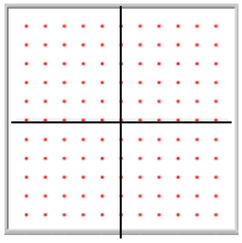
Find the slope of the line between each pair of points. Use your geoboard with a rubber band to model each line, then form a right triangle to find the rise and the run. Sketch each model.

3) $(-3,0)$ & $(1,5)$ 4) $(-2,-4)$ & $(0,3)$ 5) $(-1,2)$ & $(4,-1)$ 6) $(-3,-2)$ & $(-2,-5)$



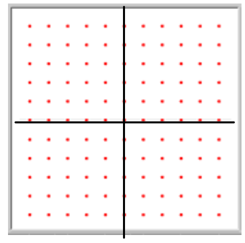
Slope= $\frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$



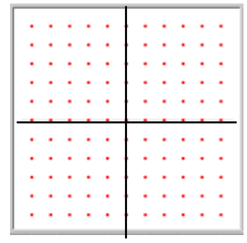
Slope= $\frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$



Slope= $\frac{\text{rise}}{\text{run}}$

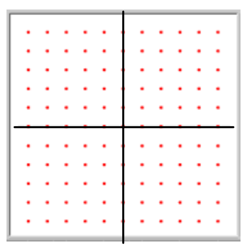
$m = \frac{\square}{\square}$



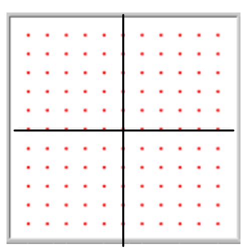
Slope= $\frac{\text{rise}}{\text{run}}$

$m = \frac{\square}{\square}$

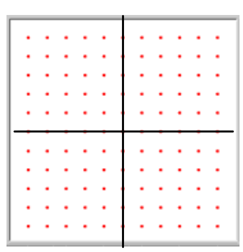
7) Start at the point $(-1,-1)$ and make a line with slope $\frac{3}{2}$ by counting the rise (up 3) and the run (over 2). Draw the line here:



8) Start at the point $(2,1)$ and make a line with slope $\frac{-1}{3}$ by counting the rise (down 1) and the run (over 3). Draw the line here:



9) Start at the point $(4,4)$ and make a line with slope $\frac{3}{4}$ by counting the rise and the run. Draw the line here:



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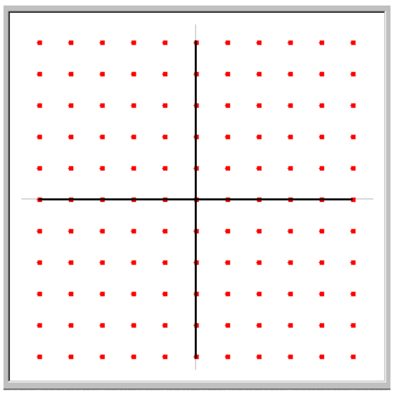
Slope of Lines Between Two Points – Extra Practice

Draw the line between each pair of points and then find its slope. You may wish to sketch a right triangle for each line to help you count the rise and the run.

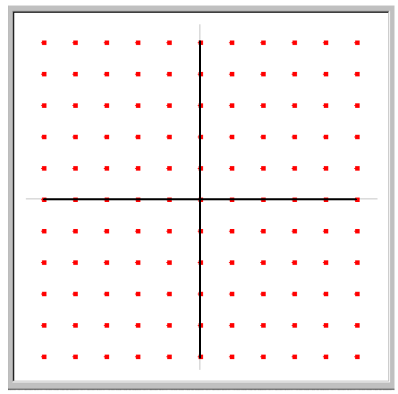
You may want to use the interactive geoboards at the website http://nlvm.usu.edu/en/nav/frames_asid_303_g_4_t_3.html?open=activities&hidepanel=true&from=topic_t_3.html.

1) $(-4, 0)$ and $(0, 5)$



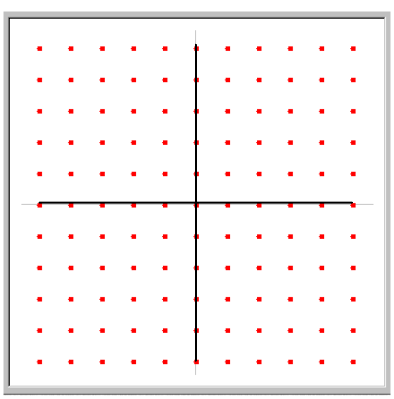
slope = ____________

2) $(0, -3)$ and $(2, 0)$



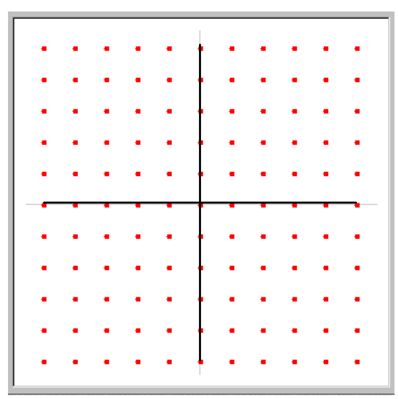
slope = ____________

3) $(-2, -3)$ and $(1, 1)$



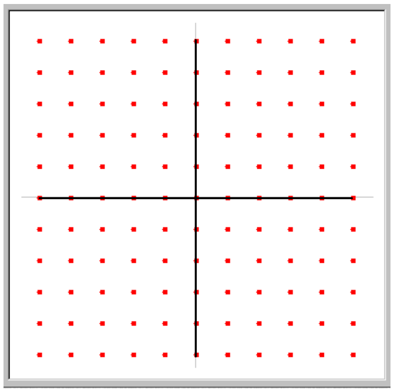
slope = ____________

4) $(-5, 2)$ and $(4, 3)$



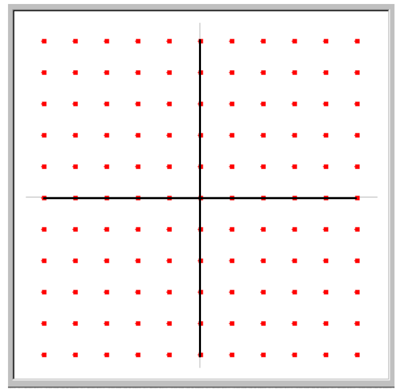
slope = ____________

5) $(-1, 4)$ and $(5, -3)$



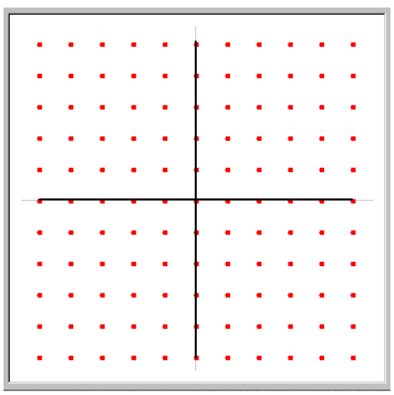
slope = ____________

6) $(-4, -2)$ and $(4, -5)$



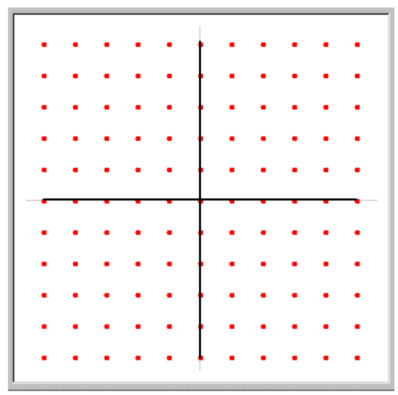
slope = ____________

7) $(-3, 2)$ and $(1, 2)$



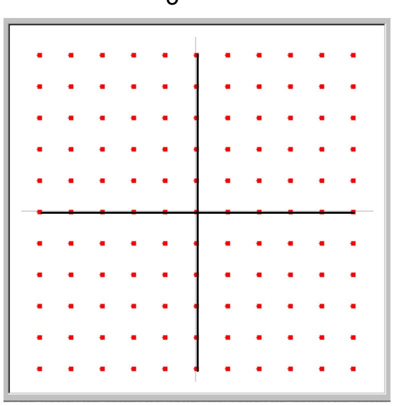
slope = ____________

8) $(5, -5)$ and $(5, 3)$

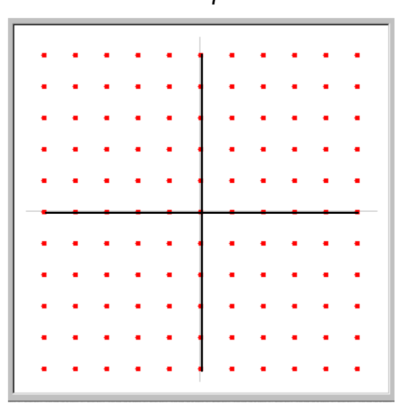


slope = ____________

9) Starting at $(-4, -3)$ sketch a line with slope $\frac{5}{3}$.



10) Starting at $(-2, 5)$ sketch a line with slope $\frac{-9}{7}$.



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Geoboard Template